

NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR

Department of Mathematics

Autumn Semester 2020-2021

SEMESTER-I	Course Handout	Date: 25/10/2020
Course No	: MA403	
Course title	: Probability and Statistics	Credit-4(3-1-0)
Instructor In-charge	: Dr. Mahendra Kumar Gupta	
Branch	: MSc Maths 1st year	

Syllabus

Definitions of Probability, Addition rule, multiplication rule, and conditional probability,

Total probability, Bayes' theorem and independence,

Discrete, continuous and mixed random variables,

Probability mass (for discrete), probability density (for continuous) and cumulative distribution functions (pmf, pdf, cdf)

Mathematical expectation, moments, moment generating function, Chebyshev's inequality,

Probability Distributions:

Discrete uniform, Binomial, Geometric, Poisson, Exponential, Gamma, Normal Distribution.

Functions of random variable, joint marginal and conditional distributions, product moments, correlation, independence of random variables, bivariate normal distribution

Central limit theorem, distributions of the sample mean and the sample variance for a normal population, Chi-Square, t- and F- distributions, The method of moments and the method of maximum likelihood estimation (MLE), confidence intervals for the mean(s) and variance (s) of normal populations, Null and alternative hypotheses, the critical and acceptance regions, two types of error, power of the test, the most powerful test and Neyman-Pearson fundamental Lemma, tests for one sample problems for normal populations.

Text Books

- Introduction to Mathematical Statistics by RV. Hogg, J McKean, AT Craig
- An Introduction to Probability and Statistics by Vijay K Rahatgi, AK Md E. Saleh

Reference Books

- Fundamentals of Statistics (Vol. 1 and 2) by AM Gun, MK Gupta, B. Dasgupta
- Fundamentals of Mathematical Statistics by SC Gupta and VK Kapoor

Evaluation Scheme:

EC No	Evaluation Component	Duration	Marks	Date and Time
1.	Mid sem. Exam	2 hours	30	As per Academic Calendar
2.	End Sem. Exam	3 hours	50	As per Academic Calendar
3.	2 Class test	30 min	5+5	During Theory Classes
4.	Assignments		5	Take home
5.	Attendance, discipline and responses in the class		5	Theory Classes

Chamber Consultation hour: 4PM -6PM for all working days.

For other timings book an appointment at mkgupta.math@nitjsr.ac.in

Notices: All notices regarding the course will be sent to given email IDs of the students

Other important rules for this course

1. **Attendance:** Students are allowed for maximum of 2 absences for the whole semester without marks deduction. After that 1 mark will be deducted for each three absences.
2. **Mass bunk:** 1 mark will be deducted for one mass bunk. Even if on a particular day, coincidentally more than 70% of students are absent, then it will be considered as a planned mass bunk and it is punishable in terms of marks.
3. **Discipline:** Maintaining discipline is mandatory in the classroom. Violation of it is punishable without explanations.
4. **Assignments:** Assignments will be given throughout the semester. You have to make a separate notebook for assignments. Every time assignment is given to you, you have to submit it on the given date and time. After checking assignments, notebook will be returned to you. After the completion of the semester, students can keep their assignment notebook. Students are encouraged to develop solution techniques of their own. It is preferred to solve questions with the methods which are not taught in the class. In that case, small mistakes would be ignored. Students are not allowed to copy each other's assignments. However, discussion among students is encouraged.
5. **Class response:** Students should be prompted to response whenever questions are asked in the classroom. They should also approach to teacher with independent proofs of the methods, challenging ideas and alternative solution techniques. Students having good interaction with the teacher would be given weightage in internal marks and grace marks.

Roll No. :

National Institute of Technology Jamshedpur
Department of Mathematics
Class test-1

M.Sc. (Maths) 1st Semester
Time Limit: 40 min

Maximum Marks: 5

MA3103 : Probability and Statistics
Instructor: Dr. Mahendra Kumar Gupta

Instructions: All questions are compulsory. All parts of a question should preferably be answered at one place. Assume missing data suitably.

1. A bowl contains 10 chips, of which 8 are marked \$2 each and 2 are marked \$5 each. Let a person choose, at random and without replacement, three chips from this bowl. If the person is to receive the sum of the resulting amounts, find his expectation. [2.5 Marks]

2. Hunters A and B shoot at a target; the probabilities of hitting the target are p_1 and p_2 , respectively. Assuming independence, can p_1 and p_2 be selected so that [2.5 Marks]

$$P(\text{zero hits}) = P(\text{one hit}) = P(\text{two hits})?$$

National Institute of Technology Jamshedpur
Department of Mathematics

Mid-Semester Examination, Sept 11, 2019

M.Sc. (Maths) 1st Semester
 Time Limit: 02 Hours

Maximum Marks: 30

MA3103 : Probability and Statistics
 Instructor: Dr. Mahendra Kumar Gupta

Instructions: All questions are compulsory. All parts of a question should preferably be answered at one place. Assume missing data suitably.

1. Consider the events C_1, C_2 , and C_3 . If $p(C_1) = 4/10$, $p(C_2) = 3/10$, $p(C_3) = 5/10$. Are C_1, C_2, C_3 mutually exclusive? Justify your answer. [2 Marks]
2. A bowl contains 16 chips, of which 6 are red, 7 are white, and 3 are blue. If 4 chips are taken at random and without replacement, find the probability that: (a) each of 4 chips is red. (b) none of 4 chips is red. (c) there is at least one chip of each color. [1.5+1.5+3 Marks]
3. A bowl contains 8 chips, 3 are red, 5 are blue. 2 chips are to be drawn successively, at random and without replacement. Compute the probability that the first draw result is a red chip and second draw result is a blue chip. [3 Marks]
4. At the beginning of a study of individuals, 15% were classified as heavy smokers, 30% were classified as light smokers, and 55% were classified as nonsmokers. In the five-year study, it was determined that the death rates of the heavy and light smokers were 5 and 3 times that of the nonsmokers, respectively. A randomly selected participant died over the 5-year period: calculate the probability that the participant was a nonsmoker. [5 Marks]
5. Suppose X has the pdf

$$f_X(x) = \begin{cases} cx^3 & 0 < x < 2, \\ 0 & \text{elsewhere} \end{cases}$$
 Calculate the value of c . Compute the probability $P(1/4 < X < 1)$. [3 Marks]
6. Let random variable X is equal the number of heads in 4 independent flips of a coin. Determine the pmf of X and compute the probability that X is equal to an odd number. [3 Marks]
7. (a) In CSIR-NET exam, there are four options for each question, out of which only one option is correct. 2 marks are given for the right answer and 0.5 mark (25% negative marking) deducted for the wrong answer. A student attempts a question and answers one option at random. Find the Expectation of marks and conclude whether a student should answer at random or not if he/she has no idea about the answer.
 (b) In GATE exam, out of four options only one option is correct. A student obtains 2 marks for the right answer and loses $\frac{2}{3}$ mark (33.33% negative marking) for the wrong answer. Find the Expectation of marks for one question and conclude whether a student should answer at random or not if he/she has no idea about the answer. [2.5+2.5 Marks]
8. Let $p(x) = (\frac{1}{2})^x, x = 1, 2, 3, \dots$, zero elsewhere, be the pmf of the random variable X . Find the mgf, the mean, and the variance of X . [3 Marks]

Roll No. :

National Institute of Technology Jamshedpur
Department of Mathematics

Quiz-2, 25-11-2019

M.Sc. (Maths) 1st Semester

Time Limit: 30 min

Maximum Marks: 5

MA3103 : Probability and Statistics

Instructor: Dr. Mahendra Kumar Gupta

Instructions: All questions are compulsory. All parts of a question should preferably be answered at one place. Assume missing data suitably.

1. Let Y be the number of successes throughout n independent repetitions of a random experiment with probability of success $p = 1/4$. Determine the smallest value of n so that $P(1 \leq Y) \geq 0.70$. [2.5]
2. Let X have the pdf $f(x) = \begin{cases} \frac{1}{2\sqrt{3}} & -\sqrt{3} < x < \sqrt{3} \\ 0 & \text{elsewhere.} \end{cases}$. Then using Chebyshev's inequality find out the upper bound of this probability. [2.5]

Good Luck

National Institute of Technology Jamshedpur
Department of Mathematics
 End-Semester Exam

M.Sc. (Maths) 1st Semester
 Time Limit: 3 Hours

Maximum Marks: 50

MA3103 : Probability and Statistics
 Instructor: Dr. Mahendra Kumar Gupta

1. A bowl contains eight chips. Three of the chips are red and the remaining five are blue. Two chips are to be drawn successively, at random and without replacement. Compute the probability that first draw is a red chip and second draw is a blue chip. [3]

2. Let X have the pdf

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the pdf of the random variable $Y = -2 \log X$. [5]

3. Let X_1 and X_2 have the pdf

$$f(x_1, x_2) = \begin{cases} 8x_1x_2 & 0 < x_1 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate $E(X_1X_2^2)$ and $E(X_2)$ and hence calculate $E(7X_1X_2^2 + 5X_2)$ [5]

4. Let $f_{1|2}(x_1|x_2) = \frac{c_1x_1}{x_2^2}$, $0 < x_1 < x_2$, $0 < x_2 < 1$, zero elsewhere be the conditional pdf of X_1 , given $X_2 = x_2$ and $f_2(x_2) = c_2x_2^4$, $0 < x_2 < 1$, zero elsewhere, be the marginal pdf of X_2 . Determine:

(a) The constants c_1 and c_2 .

(b) The joint pdf of X_1 and X_2 .

(c) $P(\frac{1}{4} < X_1 < \frac{1}{2})$. [6]

5. Let X and Y have the joint pmf described as follows:

(x,y)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
p(x,y)	2/15	4/15	3/15	1/15	1/15	4/15

and $p(x, y) = 0$ elsewhere. Find means μ_1 and μ_2 , the variances σ_1^2 and σ_2^2 , and the correlation coefficient ρ . [6]

6. Let the independent random variables X_1 and X_2 have binomial distribution with parameters $n_1 = 3, p = \frac{2}{3}$ and $n_2 = 4, p = \frac{1}{2}$, respectively. Compute $P(X_1 = X_2)$. [5]

7. In a manuscript, only 13.5% of the pages contain no typing errors. Assume that the number of errors per page is a random variable with a Poisson distribution, find the percentage of pages that have exactly 1 error. [5]

8. Calculate the mgf of $\Gamma(\alpha, \beta)$ distribution. Find out its expectation μ and variance σ^2 . [5]

9. Show that integration of pdf of general normal distribution on real line is 1 *i.e.* [3]

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} dx = 1$$

10. Let X be one or zero if, respectively, the outcome of a Bernoulli experiment is success or failure. Let $\theta, 0 < \theta < 1$, denote the probability of success. Write pmf $p(x; \theta)$. If X_1, X_2, \dots, X_n is a random sample on X , then estimate θ using the maximum likelihood function (MLE). [4]

11. A random sample of 50 items gives the mean 6.2 and variance 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance. [3]

Good Luck