Matrix Inversion by Triangualarization method

Lecture-13

Let $A = LU$

where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ → Lower

$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$ → upper

are two triangular matrices.

Then $A^{-1} = [LU]^{-1} = U^{-1}L^{-1}$

Q. find the Inverse of $A = \begin{bmatrix} 50 & 107 & 36 \\ 25 & 54 & 20 \\ 31 & 66 & 21 \end{bmatrix}$

Let $A = LU$

Then. $\begin{bmatrix} 50 & 107 & 36 \\ 25 & 54 & 20 \\ 31 & 66 & 21 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$

$$\begin{bmatrix} 50 & 107 & 36 \\ 25 & 54 & 20 \\ 31 & 66 & 21 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12}+U_{22} & l_{21}U_{13}+U_{23} \\ l_{31}U_{11} & l_{31}U_{12}+l_{32}U_{22} & l_{31}U_{13}+l_{32}U_{23}+U_{33} \end{bmatrix}$$

$\Rightarrow$ $U_{11} = 50$

$U_{12} = 107$

$U_{13} = 36$

$l_{21}U_{11} = 25$

$l_{21} = 25/50 = 1/2$

$l_{21}U_{12} + U_{22} = 54$

$\frac{1}{2} \times 107 + U_{22} = 54$

$U_{22} = 54 - \frac{107}{2} = \frac{1}{2}$

$l_{21} U_{13} + U_{23} = 20$

$\frac{1}{2} \times 36 + U_{23} = 20$

$U_{23} = 20 - 18 = 2$

$l_{31} U_{11} = 31$

$l_{31}(50) = 31$

$l_{31} = \frac{31}{50}$

$l_{31} U_{12} + l_{32} U_{22} = 66 \Rightarrow \frac{31}{50} \times 107 + l_{32}\left(\frac{1}{2}\right) = 66$

$l_{32} = 2\left[66 - \frac{31 \times 107}{50}\right] = -\frac{17}{25}$

$l_{31} U_{13} + l_{32} U_{23} + U_{33} = 21$

$\Rightarrow \frac{31}{50} \times 36 + \left(\frac{-17}{25}\right) \times 2 + U_{33} = 21$

$(31 \times 36) - (17 \times 4) + 50 U_{33} = 50 \times 21$

$1116 - 68 + 50 U_{33} = 1050$

$50 U_{33} = 1050 - 1116 + 68 = 2$

$U_{33} = \frac{1}{25}$

Thus.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 31/50 & -17/25 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 50 & 107 & 36 \\ 0 & 1/2 & 2 \\ 0 & 0 & 1/25 \end{bmatrix}$$

Now we need to find the inverse of L & U seperately

Let $L^{-1} = X$

where X is lower Triangular matrix.

Then $LX = I$

Let $X = \begin{bmatrix} x_{11} & 0 & 0 \\ x_{21} & x_{22} & 0 \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$

$LX = I \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 31/50 & -17/25 & 1 \end{bmatrix} \begin{bmatrix} x_{11} & 0 & 0 \\ x_{21} & x_{22} & 0 \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x_{11} & 0 & 0 \\ \frac{1}{2}x_{11} + x_{21} & x_{22} & 0 \\ \frac{31}{50}x_{11} - \frac{17}{25}x_{21} + x_{31} & -\frac{17}{25}x_{22} + x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\underline{\underline{x_{11} = 1}}$

$\underline{\underline{x_{22} = 1}}$

$\underline{\underline{x_{33} = 1}}$

$\frac{1}{2}x_{11} + x_{21} = 0$

$\frac{1}{2} + x_{21} = 0$

$\underline{\underline{x_{21} = -1/2}}$

$\frac{31}{50}x_{11} - \frac{17}{25}x_{21} + x_{31} = 0$

$\frac{31}{50} + \frac{17}{50} + x_{31} = 0$

$\frac{48}{50} + x_{31} = 0 \Rightarrow x_{31} = \underline{\underline{-\frac{24}{25}}}$

$$\frac{-17}{25}x_{22} + x_{32} = 0$$

$$\frac{-17}{25} + x_{32} = 0$$

$$x_{32} = \frac{17}{25}$$

Hence.

$$X = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -48/50 & 17/25 & 1 \end{bmatrix}$$

$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -24/25 & 17/25 & 1 \end{bmatrix}$$

similarily $$U^{-1} = \begin{bmatrix} 1/50 & -107/25 & 196 \\ 0 & 2 & -100 \\ 0 & 0 & 25 \end{bmatrix}$$

Hence.

$$A^{-1} = U^{-1}L^{-1}$$

$$= \begin{bmatrix} 1/50 & -107/25 & 196 \\ 0 & 2 & -100 \\ 0 & 0 & 25 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -24/25 & 17/25 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -186 & 129 & 196 \\ 95 & -66 & -100 \\ -24 & 17 & 25 \end{bmatrix}$$

Q. find the inverse of matrix.

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & -2 & -1 \end{bmatrix}$$

by factorization method.

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -3 & 5 \\ 1 & -5 & 7 \\ -1 & 7 & -11 \end{bmatrix}$$