Ques. find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using Gaussian method.

Sol. Let $X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$ be the inverse of A.

$\therefore$ $AX = I$ where I is the 3X3 unit matrix

The augmented system can be written as -

$$A = \left[\begin{array}{ccc:ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right]$$

Apply $R_2 \to R_2 - \frac{3}{2} R_1$

$R_3 \to R_3 - \frac{1}{2} R_1$

$$A \sim \left[\begin{array}{ccc:ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1/2 & 3/2 & -3/2 & 1 & 0 \\ 0 & 7/2 & 17/2 & -1/2 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - 7R_2$

$$A \sim \left[\begin{array}{ccc:ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1/2 & 3/2 & -3/2 & 1 & 0 \\ 0 & 0 & -2 & 10 & -7 & 1 \end{array}\right]$$

Thus the equation $AX = I$ is equivalent to the following 3 systems. —

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & 3/2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \\ x_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ -3/2 \\ 10 \end{bmatrix} \quad -(1)$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & 3/2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \\ x_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix} \quad -(2)$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1/2 & 3/2 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_{13} \\ x_{23} \\ x_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad -(3)$$

we solve the equations (1), (2) and (3) separately.

from (1) we get -

$$2x_{11} + x_{21} + x_{31} = 1$$

$$\frac{1}{2}x_{21} + \frac{3}{2}x_{31} = -\frac{3}{2}$$

$$-2x_{31} = 10$$

By backward substitution we get -

$$x_{31} = -5$$

$$\frac{1}{2}x_{21} + \frac{3}{2}(-5) = -\frac{3}{2}$$

$$\frac{1}{2}x_{21} = -\frac{3}{2} + \frac{15}{2} = 6$$

$$x_{21} = 12$$

and $2x_{11} + 12 - 5 = 1$

$\Rightarrow$ $2x_{11} = -6$

$$x_{11} = -3$$

Hence $x_{11} = -3$, $x_{21} = 12$ & $x_{31} = -5$

Similarly from (2) we get -

$$2x_{12} + x_{22} + x_{32} = 0$$

$$\frac{1}{2}x_{22} + \frac{3}{2}x_{32} = 1$$

$$-2x_{32} = -7$$

By Backward substitution.

$$x_{32} = \frac{7}{2}$$

$$x_{22} = \frac{-17}{2}$$

$$x_{12} = 5/2$$

from (3) we get -

$$2x_{13} + x_{23} + x_{33} = 0$$

$$\frac{1}{2}x_{23} + \frac{3}{2}x_{33} = 0$$

$$-2x_{33} = 1$$

By Backward substitution we get -

$$x_{33} = -1/2$$

$$x_{23} = 3/2$$

$$x_{13} = -1/2$$

Hence putting all values of $x_{11}, x_{12} \ldots x_{33}$

the Inverse of A i.e. A^{-1} is -

$$X = \begin{bmatrix} -3 & 5/2 & -1/2 \\ 12 & -17/2 & 3/2 \\ -5 & 7/2 & -1/2 \end{bmatrix}$$

Q. find the inverse of following matrix by Gauss jordon method.

$$A = \begin{bmatrix} 50 & 107 & 36 \\ 25 & 54 & 20 \\ 31 & 66 & 21 \end{bmatrix}$$

A. write $A = IA$

$$\begin{bmatrix} 50 & 107 & 36 \\ 25 & 54 & 20 \\ 31 & 66 & 21 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow \frac{1}{50} R_1$$

$$\begin{bmatrix} 1 & 107/50 & 36/50 \\ 25 & 54 & 20 \\ 31 & 66 & 21 \end{bmatrix} = \begin{bmatrix} 1/50 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 25 R_1$$

$$R_3 \rightarrow R_3 - 31 R_1$$

$$\begin{bmatrix} 1 & 107/50 & 36/50 \\ 0 & 1/2 & 2 \\ 0 & -17/50 & -66/50 \end{bmatrix} = \begin{bmatrix} 1/50 & 0 & 0 \\ -1/2 & 1 & 0 \\ -31/50 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow 2 R_2$$

$$\begin{bmatrix} 1 & 107/50 & 36/50 \\ 0 & 1 & 4 \\ 0 & -17/50 & -66/50 \end{bmatrix} = \begin{bmatrix} 1/50 & 0 & 0 \\ -1 & 2 & 0 \\ -31/50 & 0 & 1 \end{bmatrix} A$$

$$R_3 \rightarrow R_3 + \left(\frac{17}{50}\right) R_2$$

$$\begin{bmatrix} 1 & 107/50 & 36/50 \\ 0 & 1 & 4 \\ 0 & 0 & 2/50 \end{bmatrix} = \begin{bmatrix} 1/50 & 0 & 0 \\ -1 & 2 & 0 \\ -48/50 & 34/50 & 1 \end{bmatrix} A$$

$$R_1 \to R_1 - \left(\frac{107}{50}\right) R_2$$

$$\begin{bmatrix} 1 & 0 & -392/50 \\ 0 & 1 & 4 \\ 0 & 0 & 2/50 \end{bmatrix} = \begin{bmatrix} 108/50 & -214/50 & 0 \\ -1 & 2 & 0 \\ -48/50 & 34/50 & 1 \end{bmatrix} A$$

$$R_3 \to 25 R_3$$

$$\begin{bmatrix} 1 & 0 & -392/50 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 108/50 & -214/50 & 0 \\ -1 & 2 & 0 \\ -24 & 17 & 25 \end{bmatrix} A$$

$$R_1 \to R_1 + \left(\frac{392}{50}\right) R_3$$

$$R_2 \to R_2 - 4R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -186 & 129 & 196 \\ 95 & -66 & -100 \\ -24 & 17 & 25 \end{bmatrix} A$$

$\Rightarrow$ $I = BA$ so that

$$B = A^{-1} = \begin{bmatrix} -186 & 129 & 196 \\ 95 & -66 & -100 \\ -24 & 17 & 25 \end{bmatrix}$$