– Solution of Simultaneous Linear Equations –

GAUSS ELIMINATION METHOD :-

Q.1.

$$5x - y - 2z = 142$$
$$x - 3y - z = -30$$
$$2x - y - 3z = 5$$

After interchanging the first two equations, the system of equations can be written as -

$$x - 3y - z = -30 \quad — (1)$$
$$5x - y - 2z = 142 \quad — (2)$$
$$2x - y - 3z = 5 \quad — (3)$$

The variable x can be eliminated from the second and third equations by performing

$$2 \rightarrow (2) - 5\times(1)$$
$$3 \rightarrow (3) - 2\times(1)$$

So $(2) - 5\times(1) \Rightarrow (5x - y - 2z) - 5(x - 3y - z) = 142 - (-30\times5)$

$$\Rightarrow \cancel{5x} - y - 2z - \cancel{5x} + 15y + 5z = 142 + 150$$

$$\Rightarrow 14y + 3z = 292$$

further,

$(3) - 2\times(1) \Rightarrow (2x - y - 3z) - 2(x - 3y - z) = 5 - (-30\times2)$

$$\Rightarrow \cancel{2x} - y - 3z - \cancel{2x} + 6y + 2z = 5 - 60$$

$$\Rightarrow 5y - z = 55$$

Thus new system became -

$$x - 3y - z = -30 \qquad - (4)$$

$$14y + 3z = 292 \qquad - (5)$$

$$5y - z = 65 \qquad - (6)$$

Now we need to eliminate y from eq^n 5 & 6

This we can do by performing

$(6) \rightarrow \quad 14 \times (6) - 5 \times (5)$ operation

Thus

$$14 \times (6) - 5 \times (5) \Rightarrow \quad 14(5y - z) - 5(14y + 3z) = 14 \times 65 - 292 \times 5$$

$$\Rightarrow \cancel{70y} - 14z - \cancel{70y} - 15z = 910 - 1460$$

$$\Rightarrow \quad -29z = -550$$

$$\Rightarrow \quad z = \frac{550}{29} = 18.96$$

when $z = 18.96$

from Equation (5)

$$14y + 3(18.96) = 292$$

$$14y + 56.88 = 292$$

$$14y = 292 - 56.88$$

$$= 235.12$$

$$y = 235.12/14 = 16.79$$

Then from eq^n (4)

$$x - 3(16.79) - 18.96 = -30$$

$$x - 50.37 - 18.96 = -30$$

$$x = 50.37 + 18.96 - 30 = 39.33$$

Thus. $x = 39.33$, $y = 16.79$, $z = 18.96$

Q. Solve.

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

Solve by Gauss elimination method.

Ans. $x = \frac{35}{18} = 1.94$

$y = \frac{29}{18} = 1.61$

$z = \frac{5}{18} = 0.28.$

- **Gauss-Jordon method :-**

Q.1. Apply Gauss - Jordon's method to solve -

$$x + 2y + z = 8$$
$$2x + 3y + 4z = 20$$
$$4x + 3y + 2z = 16$$

Sol. The given equations can be written in the matrix form as follows -

- $$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 20 \\ 16 \end{bmatrix}$$ form $AX = B$

performing.

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 4R_1$

matrix takes the form

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ -16 \end{bmatrix}$$

Again: Rewrite as -

$R_2 \to (-1) R_2$

$R_3 \to (-1) R_3$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 16 \end{bmatrix}$$

$R_3 \to R_3 - 5R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 36 \end{bmatrix}$$

whose coficient matrix is an upper triangular matrix

further:

$R_3 \to \frac{1}{12} R_3$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 3 \end{bmatrix}.$$

Again

$R_1 \to R_1 - R_3$

~~$R_2 \to R_2 + 2R_3$~~ . and $R_2 \to R_3 + 2R_3$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$$

~~$R_2 \to R_2 + 2R_3$~~

$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Thus $x = 1$
$y = 2$
$z = 3$

Que. solve the following system of eqn by

(i) Gauss elimination method

(ii) Gauss jordan method.

$$x + y + z = 9$$

$$2x - 3y + 4z = 13$$

$$3x + 4y + 5z = 40$$

Answers $\Rightarrow$ $x = 1$
$y = 3$
$z = 5$