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B.TECH (3rd Semester) **ENGINEERING MATHEMATICS – III(MA1301)**

TUTORIAL SHEET-1

1. Expand $f(x) = x\sin x, 0 < x < 2\pi$ in Fourier series.
2. Find the Fourier series for $f(x) = \pi + x$ in $(-\pi, \pi)$.
3. Obtain the Fourier series for the following functions:
   I. $f(x) = -5x + 2$ in the interval $[-\pi, \pi]$.
   II. $f(x) = x - \frac{1}{2}$ in the interval $[-\pi, \pi]$.
   III. $f(x) = 2x^2 - 3$ in the interval $[0, 2\pi]$.
   IV. $f(x) = -x^2 + \frac{1}{3}$ in the interval $[0, 2\pi]$.
   V. $f(x) = x - x^2$ in the interval $[-\pi, \pi]$.
   VI. $f(x) = x$ in the interval $0 < x < 2\pi$.
4. Using the Fourier series of e^{-ax} over the interval $(-\pi, \pi)$ find the value of the series $\frac{1}{2^2+1} - \frac{1}{3^2+1} + \frac{1}{4^2+1} - \ldots\ldots\ldots\ldots\ldots\ldots$

5 Obtain the Fourier series of $f(x) = x + x^2$ in the interval $(-\pi, \pi)$. Hence prove that $\sum_{1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

6 Obtain the Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2, 0 < x < 2\pi$. Hence obtain the following relations:

   I. $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{6}$.
   II. $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$.

III. $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..........=\frac{\pi^2}{8}.$

7. Obtain the Fourier series to represent $f(x)=e^{-x}, 0< x<2\pi.$

8. Express $f(x)=|x|, -\pi<x<\pi$, as Fourier series. Hence show that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..........=\frac{\pi^2}{8}.$$

9. Obtain the Fourier series of $f(x)=x^2$ in the interval $-\pi\leq x\leq\pi$. Hence prove

that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+..........=\frac{\pi^2}{6}.$

10. Obtain the Fourier series for the function $f(x)$ given by

$$f(x)=\begin{cases}1+\frac{2x}{\pi}, -\pi\leq x\leq 0\\ 1-\frac{2x}{\pi}, 0\leq x\leq\pi\end{cases}$$

Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+..........=\frac{\pi^2}{8}.$

Answers

1. $x\sin x = -1 + \pi\sin x - \frac{1}{2}\cos x + 2\sum_{2}^{\infty}\frac{1}{n^2-1}\cos nx.$

2. $\pi + x = \frac{\pi}{2} - 2\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin nx\,.$

3. (I) $f(x) = 2 + 10\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin nx$ (II) $f(x) = -\frac{1}{2} - 2\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin nx$

(III) $f(x) = \frac{8\pi^2 - 9}{3} + 8\sum_{1}^{\infty}[\frac{\cos nx}{n} - \frac{\pi}{n}\sin nx]$ (IV) $f(x) = \frac{1-4\pi^2}{3} - 4\sum_{1}^{\infty}[\frac{\cos nx}{n} - \frac{\pi}{n}\sin nx]$

(V) $f(x) = \frac{-\pi^2}{3} + \sum_{1}^{\infty}[\frac{4(-1)^{n+1}}{n^2}\cos nx + \frac{2(-1)^{n+1}}{n}\sin nx]$ (VI) $f(x) = \pi - 2\sum_{1}^{\infty}\frac{\sin nx}{n}\,.$

4. $\pi/(2\sinh\pi)$

5. $\frac{\pi^2}{3} + 4\sum_{1}^{\infty}\frac{(-1)^n}{n^2}\cos nx - 2\sum_{1}^{\infty}\frac{(-1)^n}{n}\sin nx$

6. $\frac{\pi^2}{12} + \sum_{1}^{\infty}\frac{\cos nx}{n^2}$

7. $\frac{1-e^{-2\pi}}{\pi}[\frac{1}{2} + (\frac{1}{2}\cos x + \frac{1}{5}\cos 2x + \frac{1}{10}\cos 3x + ...) + (\frac{1}{2}\sin x + \frac{2}{5}\sin 2x + \frac{3}{10}\sin 3x + ...)]$

8. $\frac{\pi}{2} - \frac{4}{\pi}(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...)\,.$

9. $\frac{\pi^2}{3} - 4(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \frac{\cos 4x}{4^2} + ...)\,.$

10. $\frac{8}{\pi^2}(\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + ...)\,.$

<u>TUTORIAL SHEET-2</u>

1. Find Fourier expansion for the following functions:

I. $f(x) = x - x^2, \ -1 < x < 1.$

II. $f(x) = x^2 - 2$, in the interval $-2 \le x \le 2.$

III. $f(x) = e^{-x}$, in the interval $-l < x < l.$

IV. $f(x) = 1 - x^2$ in the interval $-1 \le x \le 1.$

V. $f(x) = \begin{cases} \pi x, \ 0 \le x \le 1 \\ \pi(2-x), 1 \le x \le 2 \end{cases}$

VI. $f(x) = x - x^3$, in the interval $-1 < x < 1.$

VII. $f(x) = \pi x$, in the interval $-c < x < c.$

VIII. $f(x) = \begin{cases} 0, -2 < x < 0 \\ 1, 0 < x < 2 \end{cases}$ in the interval (-2, 2).

IX. $f(x) = \begin{cases} x, 0 < x < 1 \\ 0, 1 < x < 2 \end{cases}$ in the interval (0, 2).

X. $f(x) = \begin{cases} x, 0 < x < 1 \\ 1 - x, 1 < x < 2 \end{cases}$ in the interval (0, 2).

XI. $f(x) = \begin{cases} 0, -2 < x < -1 \\ k, -1 < x < 1 \\ 0, 1 < x < 2 \end{cases}$ in the interval (-2, 2).

2. A sinusoidal voltage $E \sin \omega t$ is passed through a half wave rectifier, which clips the negative portion of the wave. Expand the resulting periodic function in a Fourier series where

$$f(t) = \begin{cases} 0, \ \dfrac{-T}{2} < t < 0 \\ E \sin \omega t, 0 < t < \dfrac{T}{2} \end{cases} \quad \text{and } T = \frac{2\pi}{\omega}.$$

3. Find the Fourier coefficients corresponding to the function $f(t) = \begin{cases} 0, \ -5 < t < 0 \\ 3, \ 0 < t < 5 \end{cases}$

4. Obtain the Fourier series for the square wave up to four terms as discuss in lecture of LT.

5. Determine the first four terms of the Fourier series for the triangular waveform discuss in lecture of LT.

6. Find the first four terms of the Fourier series for the saw tooth waveform discuss in lecture of LT.

Answers

1. I $\frac{-1}{3}+\frac{4}{\pi^2}(\frac{\cos\pi x}{1^2}-\frac{\cos 2\pi x}{2^2}+\frac{\cos 3\pi x}{3^2}-...........)+\frac{2}{\pi}(\frac{\sin\pi x}{1}-\frac{\sin 2\pi x}{2}+\frac{\sin 3\pi x}{3}-...........)$

II $\frac{-2}{3}-\frac{16}{\pi^2}(\cos\frac{\pi x}{2}-\frac{1}{4}\cos\pi x+\frac{1}{9}\cos\frac{3\pi x}{2}-..........)$.

III $\sinh l\left[\frac{1}{l}+2l\sum_{n=1}^{\infty}\frac{(-1)^n}{l^2+n^2\pi^2}\cos\frac{n\pi x}{l}+2\pi\sum_{n=1}^{\infty}\frac{n(-1)^n}{l^2+n^2\pi^2}\sin\frac{n\pi x}{l}\right]$.

IV $\frac{2}{3}+\frac{4}{\pi^2}(\cos\pi x-\frac{\cos 2\pi x}{2^2}+\frac{\cos 3\pi x}{3^2}-.....)$. V $\frac{\pi}{2}-\frac{4}{\pi}(\frac{\cos\pi x}{1^2}+\frac{\cos 3\pi x}{3^2}+\frac{\cos 5\pi x}{5^2}+.....)$.

VI $\frac{12}{\pi^3}(\sin\pi x-\frac{\sin 2\pi x}{2^3}+\frac{\sin 3\pi x}{3^3}-.....)$. VII $2c[\sin\frac{\pi x}{c}-\frac{1}{2}\sin\frac{2\pi x}{c}+\frac{1}{3}\sin\frac{3\pi x}{c}-.....)$.

VIII $\frac{1}{2}+\frac{2}{\pi^2}(\sin\frac{\pi x}{2}+\frac{1}{3}\sin\frac{3\pi x}{2}+\frac{1}{5}\sin\frac{5\pi x}{2}+.....)$.

IX $\frac{1}{4}-\frac{2}{\pi^2}(\cos\pi x+\frac{\cos 3\pi x}{3^2}+\frac{\cos 5\pi x}{5^2}+....)+\frac{1}{\pi}(\sin\pi x-\frac{\sin 2\pi x}{2}+\frac{\sin 3\pi x}{3}+......)$.

X $\frac{-4}{\pi^2}(\cos\pi x+\frac{\cos 3\pi x}{3^2}+\frac{\cos 5\pi x}{5^2}+....)+\frac{2}{\pi}(\sin\pi x+\frac{\sin 3\pi x}{3}+...)$

XI $\frac{k}{2}+\frac{2k}{\pi}(\cos\frac{\pi x}{2}-\frac{1}{3}\cos\frac{3\pi x}{2}+\frac{1}{5}\cos\frac{5\pi x}{2}+...)$.

2. $\frac{E}{\pi}+\frac{E}{2}\sin\omega t-\frac{2E}{\pi}(\frac{\cos 2\omega t}{1.3}+\frac{\cos 4\omega t}{3.5}+\frac{\cos 6\omega t}{5.7}+...........)$.

3. $a_0=3,\ a_n=0,\ b_n=\begin{cases}0, & if\ n\ is\ even\\ \frac{6}{n\pi}, & if\ n\ is\ odd\end{cases}$

4. $\frac{a}{2}+\frac{2a}{\pi}[\sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x+\frac{1}{7}\sin 7x+...........]$.

TUTORIAL SHEET-3

1. Express $f(x) = x$ as a sine series in $0 < x < \pi$.

2. Find the Fourier sine series expansion of the following functions:

I. $f(x) = \begin{cases} x, \ 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}, \ \frac{\pi}{2} \le x < \pi \end{cases}$ II. $f(x) = e^{ax}$ for $0 < x < \pi$ III. $f(x) = 2x - 1$ for $0 < x < 1$.

3. Represent the following function by a Fourier cosine series:

I. $f(x) = \begin{cases} 1, 0 < x < \frac{\pi}{2} \\ 0, \frac{\pi}{2} < x < \pi \end{cases}$ II. $f(x) = \sin\frac{\pi x}{l}, 0 < x < l$. III. $f(x) = e^x$ in the interval (0, 1).

IV. $f(x) = \begin{cases} 2x, \ 0 < x < 1 \\ 2(2-x), \ 1 < x < 2 \end{cases}$ V. $f(x) = \pi - x$ in $(0, \pi)$. VI. $f(x) = x^2$ in $(0, \pi)$

VII. $f(x) = \cos(sx), -\pi \le x \le \pi$. VIII. $f(x) = e^x$ in the interval $0 < x < \pi$.

4. Find a half range even expansion of the function $f(x) = (\frac{-x}{l}) + 1, \quad 0 \le x \le l$.

5. Find a series of cosine of multiples of *x* which will represent $f(x)$ in $(0, \pi)$ where

$f(x) = \begin{cases} 0, \ 0 < x < \frac{\pi}{2} \\ \frac{\pi}{2}, \ \frac{\pi}{2} < x < \pi \end{cases}$ and deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ... = \frac{\pi}{4}$.

6. If $f(x) = \begin{cases} x, \ 0 < x < \frac{\pi}{2} \\ \pi - x, \ \frac{\pi}{2} < x < \pi \end{cases}$ show that I. $f(x) = \frac{4}{\pi}(\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - ....)$

II. $f(x)=\frac{\pi}{4}-\frac{2}{\pi}(\frac{1}{1^2}\cos 2x+\frac{1}{3^2}\cos 6x+\frac{1}{5^2}\cos 10x+..............)$

7. If $f(x)=x+1,$ for $0<x<\pi$, find its Fourier sine series and Fourier cosine series. Hence deduce that I. $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+................=\frac{\pi}{4}$ II. $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+..............=\frac{\pi^2}{8}$.

Answers

1. $2[\sin x-\frac{1}{2}\sin 2x+\frac{1}{3}\sin 3x-.........]$.

2. I. $\left(\frac{2}{\pi}+1\right)\sin x-\frac{1}{2}\sin 2x+\left(\frac{-2}{9\pi}+\frac{1}{3}\right)\sin 3x+...$ II.

$\frac{2}{\pi}[\frac{1+e^{a\pi}}{a^2+1}\sin x+\frac{2(1-e^{a\pi})}{a^2+2^2}\sin 2x+...]$.

III. $\frac{-2}{\pi}[\sin 2\pi x+\frac{1}{2}\sin 4\pi x+\frac{1}{3}\sin 6\pi x+............]$.

3. I. $\frac{1}{2}+\frac{2}{\pi}[\cos x-\frac{1}{3}\cos 3x+\frac{1}{5}\cos 5x-........]$.

II. $\frac{2}{\pi}-\frac{4}{\pi}[\frac{1}{3}\cos\frac{2\pi x}{l}+\frac{1}{15}\cos\frac{4\pi x}{l}+\frac{1}{35}\cos\frac{6\pi x}{l}+.......]$.

III. $e-1+2[\frac{-e-1}{\pi^{2+1}}\cos\pi x+\frac{e-1}{4\pi^2+1}\cos 2\pi x+\frac{-e-1}{9\pi^2+1}\cos 3\pi x+......]$.

IV. $1-(\frac{8}{\pi^2}+\frac{4}{\pi})\cos\frac{\pi x}{2}-\frac{4}{\pi^2}\cos\frac{2\pi x}{2}+(\frac{-8}{9\pi^2}+\frac{4}{3\pi})\cos\frac{3\pi x}{2}+..........$

V. $\frac{\pi}{2}+\frac{4}{\pi}(\cos x+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+..........)$.

VI $\frac{\pi^2}{3} - \frac{4}{\pi}(\cos x - \frac{1}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x - ....)$.

VII. $\frac{\sin \pi x}{\pi s} + \frac{1}{\pi}\sum[\frac{\sin(s\pi + n\pi)}{s+n} + \frac{\sin(s\pi - n\pi)}{s-n}]\cos nx$.

VIII $\frac{e^{\pi} - 1}{\pi} - \frac{2}{\pi}\sum_{1}^{\infty}\frac{1-(-1)^n e^{\pi}}{n^2+1}\cos nx$.

4. $\frac{1}{2} + \frac{4}{\pi^2}[\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \frac{1}{5^2}\cos\frac{5\pi x}{l} + ...]$ 5.

$\frac{\pi}{4} - \cos x + \frac{1}{3}\cos 3x - \frac{1}{5}\cos 5x + ...$