

Q. Show that

$$1) \int_0^{\infty} \frac{\sin \pi \omega \sin x \omega}{1-\omega^2} d\omega = \begin{cases} \frac{\pi}{2} \sin x, & 0 \leq x \leq \pi \\ 0, & x > \pi \end{cases}$$

$$2) \int_0^{\infty} \frac{\sin \omega - \omega \cos \omega}{\omega^2} \sin x \omega d\omega = \begin{cases} \frac{\pi x}{2}, & 0 < x < 1 \\ \frac{\pi}{4}, & x = 1 \\ 0, & x > 1 \end{cases}$$

Q. Find the Fourier Cosine integral representation of the following :-

$$1. f(x) = \begin{cases} x^2, & 0 < x < 1 \\ 0, & x > 1 \end{cases} \quad \textcircled{2} f(x) = \frac{1}{1+x^2} \quad (x > 0)$$

$$\textcircled{3} f(x) = \begin{cases} e^{-x}, & 0 < x < a \\ 0, & x > a \end{cases}$$

Q. Find the Fourier Sine integral of the following.

$$1) f(x) = \begin{cases} x, & 0 < x < a \\ 0, & x > a \end{cases} \quad \textcircled{2} f(x) = \begin{cases} \cos x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

Q. Show that

$$1) f(ax) = \frac{1}{a} \int_0^{\infty} A\left(\frac{\omega}{a}\right) \cos x \omega d\omega \quad (a > 0)$$

$$2) x f(x) = \int_0^{\infty} B^*(\omega) \sin x \omega d\omega, \quad B^* = -\frac{dA}{d\omega}$$

$$3) x^2 f(x) = \int_0^{\infty} A^*(\omega) \cos x \omega d\omega, \quad A^* = \left(-\frac{d^2 A}{d\omega^2}\right)$$