Department of Mathematics
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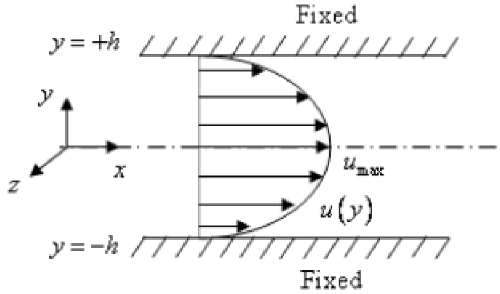
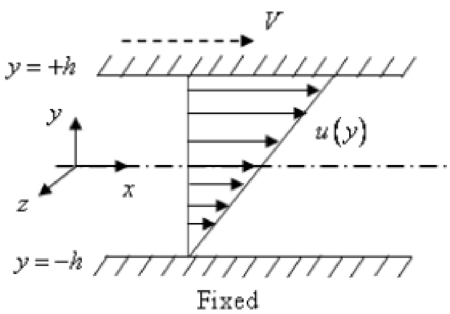
MAG6134: Fluid Dynamics
Assignment-2

Q.1 Consider a two-dimensional incompressible, viscous, laminar flow between two parallel plates, separated by certain distance $(2h)$ as shown in figure below. Both the plates are fixed but the pressure varies in $x - direction$. It is assumed that the plates are very wide and long so that the flow is essentially axial $(u \neq 0;\ v = w = 0)$. Further, the flow is considered far downstream from the entrance so that it can be treated as fully-developed.



For this flow problem, prove that the maximum velocity (u_{max}) is $1.5 - times$ the average velocity.

Q.2 Consider a two-dimensional incompressible, viscous, laminar flow between two parallel plates separated by certain distance $(2h)$ as shown in figure below. The upper plate moves with constant velocity (V) while the lower is fixed and there is no pressure gradient. It is assumed that the plates are very wide and long so that the flow is essentially axial $(u \neq 0;$ $v = w = 0)$. Further, the flow is considered far downstream from the entrance so that it can be treated as fully-developed.



Describe the velocity profile for the above problem. Find the wall shear stress and skin friction coefficient for the same.

Q.3 Plot the velocity profile for ***generalized Couette flow problem*** using MATLAB or Octave software ***(submit the graph along with the code)***.

Hint: The velocity profile is $f(\eta) = \eta + P\eta(1 - \eta)$ where $\eta = \frac{y}{h} \in [0, 1]$ and $P = -\frac{h^2}{2\mu U}\frac{dp}{dx}$ is dimensionless pressure gradient. One can plot the graph of $f(\eta)$ using ***for loop***.

Q.4 Find the volume flow rate for the ***generalized Couette flow*** between two parallel plates.

Q.5 List the important assumptions of the Stokes' first problem. Solve the IBVP obtained in ***Stokes' first problem*** using similarity transformation method.

Hint: Use $f(\eta) = \frac{u}{U}$; $\eta = \frac{y}{\sqrt{\nu t}}$ as the self-similarity variable.

Q.6 Extend the Stokes' first problem considering constant porosity at the surface of the plate.

Q.7 Find the volume flow rate for the ***Hagen-Poiseuille flow*** and show that the maximum velocity is twice that of average velocity.

Q.8 A horizontal pipe with diameter of 10 cm is used in lab to measure the viscosity of mustard oil (0.9 t/m^3). During an experiment a pressure difference of 2 t/m^3 is noted from two pressure gauges, which are located 10 m apart on the pipe. Oil is allowed to discharge into a weighing tank, and a total of 800 kg of oil is collected for a duration of 2 min. Find the viscosity of the oil.

Q.9 Consider the steady fully developed flow of a viscous, incompressible fluid between two concentric rotating cylinders with radius R_1 and R_2, where $R_1 < R_2$. The fluid flow within the annulus region is induced due to an external pressure gradient along the axial direction. Present a mathematical model describing the fluid flow model with suitable boundary conditions. Solve the model using a suitable method and obtain the velocity profile. Also, obtain shear-stress at the wall of the outer cylinder.

Q.10 Consider the steady fully developed flow of a viscous, incompressible fluid between two concentric cylinders with radius R_1 and R_2, where $R_1 < R_2$, further assuming that the flow is co-axial ($i.e.$ the only non-zero component of velocity is u_θ) and there is no pressure gradient in θ & $z - diections$. If the inner cylinder is rotating with angular velocity ω_1 while the outer cylinder is at rest, then find the value of shear stress at the wall of inner cylinder.

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