Department of Mathematics
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MAG6134 : Fluid Dynamics
Assignment 1

1. The velocity components for a two-dimensional fluid system can be given in the Eulerian system by $u = 2x + 2y + 3t,\ v = x + y + t/2$. Find the displacement of a fluid particle in the Lagrangian system.

2. For a two-dimensional flow the velocities at a point in a fluid may be expressed in the Eulerian coordinates by $u = x + y + 2t$ and $v = 2y + t$. Determine the Lagrange coordinates as functions of the initial positions $x_0$and y_0 and the time t.

3. The velocity distribution of a certain two-dimensional flow is given by $u = Ay + B$ and $v = Ct$, where $A,\ B,\ C$ are constants. Obtain the equation of the motion of fluid particles in Lagrangian method.

4. Derive the conservation of mass equation in cylindrical and spherical coordinates and obtain their special cases in case of an incompressible flow.

5. Show that in a two-dimensional incompressible steady flow field the equation of continuity is satisfied with the velocity components in rectangular coordinates given by

$$u(x, y) = \frac{k(x^2 - y^2)}{(x^2 - y^2)^2}\ ;\ \ v(x, y) = \frac{2kxy}{(x^2 - y^2)^2}$$

where k is an arbitrary constant.

6. Determine the constants $l,\ m$ and n in order that the velocity

$$q = \frac{(x + lr)i + (y + mr)j + (z + nr)k}{r(x + r)},$$

where $r = \frac{(x^2+y^2+z^2)}{2}$, may satisfy the equation of continuity for a liquid.

7. A velocity field is defined by $u = 3y^2, v = 5x, w = 0$. At point (2,4,0), compute the following: (a) velocity (b) local acceleration (c) convective acceleration.

8. The velocity for a steady, incompressible flow in the xy plane is given by $\overrightarrow{V} = \frac{A}{x}\hat{\imath} + \frac{Ay}{x^2}\hat{\jmath}$, where $A = 3m^2/s$. Obtain an equation for the streamline that passes through the point $(x, y) = (2, 6)$.

9. A steady, two-dimensional velocity field is given by $\vec{q} = (2.85 + 1.26x - 0.896y)\hat{\imath} + (3.45x + cx - 1.26y)\hat{\jmath}$. Calculate the constant c such that the flow field is irrotational.

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10. Consider a steady, two-dimensional flow field in the xy-plane whose x-component of velocity is given by

$$u = a + b(x - c)^2,$$

where a, b, and c are constants with appropriate dimensions. Of what form does the y-component of velocity need to be in order for the flow field to be incompressible?

11. Consider the steady, two-dimensional velocity field given by

$$\vec{q} = (0.5 + 0.8x)\hat{\imath} + (1.5 - 0.8y)\hat{\jmath},$$

where lengths are in units of m, time in s, and velocity in m/s. Determine if there is a stagnation point in the flow. Calculate the rate of translation, rate of rotation, linear strain rate, shear strain rate, and volumetric strain rate. Verify that this flow is incompressible.