NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR, JHARKHAND-831014

Department of Mathematics

Course HOME ASSIGNMENT for M. Sc, 2nd (EVEN/SPRING) Semester, 2019 Batch, Session: 2019-20

Course No.: MA3202 Credit: 4 Course Title.:Numerical Analysis

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1(a). Find the iterative formulae for finding $\sqrt{N}$, $(N)^{1/3}$ where N is a real number, using Newton-Raphson formula. Hence evaluate: (a) $\sqrt{32}$ (b) $(41)^{1/3}$ to four places of decimal.

1(b). Evaluate the following using Newton-Raphson method:
i) 1/18 ii) $1/\sqrt{15}$ iii) $28^{-1/4}$

1(c). Define the Rate of convergence of iterative procedures and find the Rate of convergence of Fixed point iteration method, Regula-falsi method, Newton-Raphson method and Secant method.

1(d). Show that the Regula-Falsi and Fixed-point iteration methods are linearly convergent methods.

1(e). Using Regula-falsi method, Secant method and Fixed point iteration method, find the real root of the following equations correct to three decimal places:
i) $x \sin x + \cos x = 0$ ii) $e^x = x^3 + \cos 25x$ iii) $\log x - x + 3 = 0$ iv) $3x^3 - 9x^2 + 8 = 0$

2(a). Define forward, backward and central differences by forming difference table in each case and show that any higher order differences can be expressed in terms of entries.

2(b). Show that the n-th difference of a n-th degree polynomial is constant and all higher order differences are zero.

3. Extend the following table to two more terms on either side by constructing the difference table:

x	-0.2	0.0	0.2	0.4	0.6	0.8	1.0
y	2.6	3.0	3.4	4.28	7.08	14.2	29.0

4. Evaluate :
(i) $\Delta^4[(1-x)(1-2x)(1-3x)(1-4x)]$,(h=1), (ii) $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)]$, (h=2),
where h is the interval of differencing.

5. Express $3x^4-4x^3+6x^2+2x+1$ as a factorial polynomial and find differences of all orders.

6. Obtain the function whose first difference is $2x^3+3x^2-5x+4$.

7. Find the missing values in the following table:

X	0	5	10	15	20	25
Y	6	10	--	17	--	31

8. Prove that
(i) $\Delta = \mu\,\delta + (\delta^2/2)$ (ii) $\Delta = (1/2)\delta^2 + \delta\sqrt{(1+\delta^2/4)}$
(iii) $(E^{1/2} + E^{-1/2})(1+\Delta)^{1/2} = 2+\Delta$ (iv) $hD = \log(1+\Delta) = \sinh^{-1}(\mu\delta)$ (v) $\mu^2 = 1+\delta^2/2$

9. The area A of a circle of diameter d is given for the following values:

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

Calculate the area of a circle of diameter 105, using Newton backward interpolation formula.

10. Find the number of men getting wages between Rs. 10 and 15 from the following data using Newton forward interpolation formula:

Wages in rupees	0 - 10	10 - 20	20 - 30	30 - 40
frequency	09	30	35	42

11 . The pressure p of wind corresponding to velocity v is given by the following data. Estimate p when v=25 by Newton backward interpolation formula.

v	10	20	30	40
p	1.1	2	4.4	7.9

12. Given the following values of x and y, find the value of y at x=1.23 and 1.12 using centeral difference interpolation formula

x	1.0	1.05	1.10	1.15	1.20	1.25	1.30
y	1.0000	1.0247	1.0488	1.0723	1.0954	1.1180	1.1401

13. **Derive Lagrange's interpolation formula and hence find the unique polynomial of degree 3 or less, such that**

$f(0)=1,\ f(1)=3,\ f(3)=55,\ f(5)=-10.$

14. **Using Lagrange's interpolation formula, express the function $(x^2+6x-1)/((x^2-1)(x-4)(x-6))$ as a sum of partial fractions.**

15. **Find the error associated with Polynomial interpolation formula.**

16. **Using Lagrange's inverse interpolation formula, find the root of the equation f(x)=0 , given that f(30)= -30, f(34)= -13, f(38)= 3 and f(42)= -18.**

17. **Derive Newton divided interpolation formula and find the value of f(27) , given that f(20)= 0, f(34)= -13, f(35)= 3 and f(42)= -18.**

18. **The following table gives the viscosity of an oil as a function of temperature. Use Newton divided difference interpolation formula to find the viscosity of oil at a temperature of 140^0.**

Temp.^{0}	**110**	**130**	**160**	**190**
Viscosity	**10.8**	**8.1**	**5.5**	**4.8**

19.(a) What do you mean by spline function and spline interpolation ?

(b) Write the step by step procedure of (i) Linear (ii) quadratic and (iii) cubic spline interpolation polynomial processes respectively.

(c) Obtain the (i) Linear (ii) quadratic and (iii) cubic spline interpolation polynomials respectively that passes through the points (4, -43), (7, 83), (9, 327) and (12, 1053).

20. Using maximum norm compute and interpret the condition number for the following matrices:

(i) $\begin{bmatrix} 1 & -2 \\ -0.99 & 1.99 \end{bmatrix}$ **(ii)** $\begin{bmatrix} -0.6 & 0.6 \\ 0.4 & 0.2 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$

21. Check whether the following system of equations are ill or well conditioned:

(i) $\begin{bmatrix} 1 & -2 \\ -0.99 & 1.99 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 4 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}=\begin{bmatrix} 32. \\ 23 \\ 33 \\ 31 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 1 \\ 2.01 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 2. \\ 2.05 \end{bmatrix}$

22(a). Solve the following system of equations by (i) Gaussian elimination (ii) Gauss Seidel and (iii) Jacobi methods respectively.

(a) $2x-6y+8z=24;\ 5x+4y-3z=2\ ;\ 3x+y+2z=16$

(b) $x+y+z=1;\ 4x+3y-z=6\ ;\ 3x+5y+3z=4$

(c) $5x-2y+z=4;\ 7x+y-5z=8\ ;\ 3x+7y+4z=10$

(d) $3x+2y+4z=7;\ 2x+y-2z=7\ ;\ x+3y+5z=2$

22(b). Solve any three system of nonlinear equations by Newton's method.

22(c).**Find the Rate of convergence of Gauss Seidel method.**

23. What are eigen values and eigenvectors of the matrix. Explain the (i) Power Method, (ii) Inverse Power Method, (iii) Householder method, and QR Algorithm **for finding the eigen values and the corresponding eigenvectors of a matrix.**

24. **Find the eigen values and the corresponding eigenvectors of any three self chosen real symmetric matrix using all the methods studied.**
25. **What is eigen value problem? Use power method and** Inverse Power Method **for finding the eigen values of the coefficient matrix of the all the above systems given in the question no. 22.**
26. **Evaluate** $\int_0^1 \frac{1}{1+x}dx$ **dx taking seven ordinates by applying Simpson 3/8 rule. Deduce the value of** $\log_e 2$**.**
27. **Evaluate the integrals (i)** $\int_0^{\pi/2} \sqrt{\sin x}\, dx$ **and (ii)** $\int_0^{\pi/2} \frac{x}{\sin x} dx$ **using (i) Simpson 1/3- rule and (ii) Trapezoidal rule of integration by taking eleven ordinates.**
28. **The velocity v of a particle at distance s from a point on its path is given by the table:**

s ft.	**0**	**10**	**20**	**30**	**40**	**50**	**60**
v ft./sec.	**47**	**58**	**64**	**65**	**61**	**52**	**38**

Estimate the time taken to travel 60 ft by using Simpson 1/3- rule. Compare the result with Simpson 3/8 rule.

29. **The following table gives the velocity v of a particle at time t :**

t (seconds)	**0**	**2**	**4**	**6**	**8**	**10**	**12**
v(m/sec.)	**4**	**6**	**16**	**34**	**60**	**94**	**136**

Find the distance moved by the particle in 12 seconds and also the acceleration at t=2 seconds.

30. **Evaluate the following integrals using using (i) Simpson 1/3- rule (ii) Simpson 3/8- rule and (iii) Trapezoidal rule of integration by taking seven ordinates respectively and find out which one is better.**

(i) $\int_2^4 (1+x^4)\,dx$ **(ii)** $\int_{-2}^{2} e^{-x^2}\,dx.$

31. **Find the error associated with (i) Simpson 1/3- rule (ii) Simpson 3/8- rule and (iii) Trapezoidal rule of integration respectively.**

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