NATIONAL INSTITUTE OF TECHNOLOGY, JAMSHEDPUR, JHARKHAND-831014

Department of Mathematics

Course Home Assignment out for MCA, 2nd (EVEN/SPRING) Semester, 2019 Batch, Session: 2019-20

Course No.: MA3201 Credit: 4 Course Title.: Computer Oriented Numerical Techniques/NUMERICAL METHODS.

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1. **Describe different number system in detail.**

2. **What are 1's and 2's complement of the following decimal numbers:**
(i) 124 (ii) 256 (iii) 113 (iv) -56 (v) -41 (vi) -27

3. **Obtain the following:**
(i) $(FACE)_{16}=(\ ?\ )_2$ (ii) $(101.0110)_2 = (\ ?\ )_{10}$ (iii) $(011001.001110)_2= (\ ?\ )_8$
(iv) $(11011101.0111)_2 =(\ ?\ )_{10}$
(v) $(3BF.5C)_{16} =(\ ?\ )_2$ (vi) $(3ABC)_{16} =(\ ?\ )_8$ (vii) $(39.B8)_{16} =(\ ?\ )_8$ (viii) $(56.08)_{16} =(\ ?\ )_{10}$
(ix) $(24.6)_8 =(\ ?\ )_{10}$

4. **Subtract using 8 bit mantissa word and verify the result by obtaining decimal equivalent:**
(i) $(1100110)_2$ from $(11101011)_2$ (ii) $(1101)_2$ from $(111001)_2$ (iii) $(10110100)_2$ from $(10010101)_2$
(iv) $(111)_2$ from $(100)_2$ (v) $(11011.0001)_2$ from $(11110.11)_2$ (vi) $(010111)_2$ from $(101011)_2$

5. **Add the following binary numbers and verify the result by obtaining decimal equivalent:**
(i) $(1110.1101)_2$ and $(110101.00101)_2$ (ii) $(1110.11)_2$ and $(11011011.111)_2$

6. **Define the following with example:**
(i) Absolute error (ii) Relative error (iii) Inherent error (iv) Round-off error (v) Truncation error
(vi) Chopping error (vii) Bugs and Debugging (viii) Overflow and Underflow on computer.

7. **Find the round-off error in the results of the following arithmetic operations, using four digit mantissa.**
(i) 27.65+22.20 (ii) 87.26+31.42 (iii) 1250.0 X 40.0 (iv) 3543.0+16.78 (v) 25.68 / 6.567
(vi) 456.7-1.531 (vii) 456.7-4.566

8. **Explain the concept of significant digit and show the situations where there is a loss of significant digit and magnification of errors in the result.**

9. **What are fixed and floating point representation of numbers ? Explain with example.**

10. **Discuss an example to show that the distributive low of arithmetic is not always satisfied in numerical computing.**

11. **What are induced and inherent instability? When these arises?**

12. **Define condition number. What is its significance in numerical computing?**

13. **Compute and interpret the condition number for the following functions:**

(i) $f(x) = Sinx$ **(ii)** $f(x) = \dfrac{1}{1-x}$ **(iii)** $f(x) = x^5$ **(iii)** $f(x) = x^{1/3}$

14. **Using maximum norm compute and interpret the condition number for the following matrices:**

(i) $\begin{bmatrix} 1 & -2 \\ -0.99 & 1.99 \end{bmatrix}$ **(ii)** $\begin{bmatrix} -0.6 & 0.6 \\ 0.4 & 0.2 \end{bmatrix}$ **(iii)** $\begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$

15. **Check whether the following system of equations are ill or well conditioned:**

59. $\begin{bmatrix} 1 & -2 \\ -0.99 & 1.99 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}$ **(ii)** $\begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 4 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 32. \\ 23 \\ 33 \\ 31 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 1 \\ 2.01 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2. \\ 2.05 \end{bmatrix}$

16. **Show that the following system of equations is ill-conditioned for computing the point of intersection when** m_1 **and** m_2 **are nearly equal.** $y = m_1x + c_1$; $y = m_2x + c_2$.

17. **Compute the difference of square roots of two numbers $x = 497.0$ and $x = 496.0$. Suggest an another approach by rearranging the terms to improve the result.**

18. **Explain the computational methods for estimation of the error associated with the final result.**

19. **Using Newton-Raphson method, find the root of the following equations correct to three decimal places:**
i) $x \sin x + \cos x = 0$ ii) $e^x = x^3 + \cos 25x$ iii) $\log x - x + 3 = 0$ iv) $3x^3 - 9x^2 + 8 = 0$

20. **Find the iterative formulae for finding $\sqrt{N}$, $(N)^{1/3}$ where N is a real number, using Newton-Raphson formula. Hence evaluate: (a) $\sqrt{32}$ (b) $(41)^{1/3}$ to four places of decimal.**

21. **Evaluate the following using Newton-Raphson method:**
i) 1/18 ii) $1/\sqrt{15}$ iii) $28^{-1/4}$

22. **Define the Rate of convergence of iterative procedures and find the Rate of convergence of Bisection method, Fixed point iteration method, Regula-falsi method, Newton-Raphson method and Secant method.**

23. **Show that the Regula-Falsi and Fixed-point iteration methods are linearly convergent methods.**

24. **Using Bisection method, Regula-falsi method, Secant method and Fixed point iteration method, find the real root of the following equations correct to three decimal places:**
i) $x \sin x + \cos x = 0$ ii) $e^x = x^3 + \cos 25x$ iii) $\log x - x + 3 = 0$ iv) $3x^3 - 9x^2 + 8 = 0$

25. **(i) Solve the following system of equations by (i) Gaussian elimination method and (ii) Gauss Seidel method respectively.**

(a) $2x - 6y + 8z = 24;\ 5x + 4y - 3z = 2\ ;\ 3x + y + 2z = 16$

(b) $x + y + z = 1;\ 4x + 3y - z = 6\ ;\ 3x + 5y + 3z = 4$

(c) $5x - 2y + z = 4;\ 7x + y - 5z = 8\ ;\ 3x + 7y + 4z = 10$

(d) $3x + 2y + 4z = 7;\ 2x + y - 2z = 7\ ;\ x + 3y + 5z = 2$

Refine the solutions obtained by Iterative refinement procedure in each case.

(ii) Find the Rate of convergence of Gauss Seidel method,

26. **Define forward, backward and central differences by forming difference table in each case and show that any higher order differences can be expressed in terms of entries.**

27. **Show that the n-th difference of a n-th degree polynomial is constant and all higher order differences are zero.**

28. **Extend the following table to two more terms on either side by constructing the difference table:**

x	**-0.2**	**0.0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
y	**2.6**	**3.0**	**3.4**	**4.28**	**7.08**	**14.2**	**29.0**

29. **Evaluate :**
(i) $\Delta^4[(1-x)(1-2x)(1-3x)(1-4x)]$,(h=1), (ii) $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)]$, (h=2), where h is the interval of differencing.

30. **Express $3x^4-4x^3+6x^2+2x+1$ as a factorial polynomial and find differences of all orders.**

31. **Obtain the function whose first difference is $2x^3+3x^2-5x+4$.**

32. **Find the missing values in the following table:**

X	**0**	**5**	**10**	**15**	**20**	**25**
Y	**6**	**10**	**--**	**17**	**--**	**31**

33. **Prove that**
(i) $\Delta = \mu\delta + (\delta^2/2)$ (ii) $\Delta = (1/2)\delta^2 + \delta\sqrt{(1+\delta^2/4)}$
(iii) $(E^{1/2} + E^{-1/2})(1+\Delta)^{1/2} = 2+\Delta$ (iv) $hD = \log(1+\Delta) = \sinh^{-1}(\mu\delta)$ (v) $\mu^2 = 1+\delta^2/2$

34. **Derive Lagrange's interpolation formula and hence find the unique polynomial of degree 3 or less, such that $f(0) = 1,\ f(1) = 3,\ f(3) = 55,\ f(5) = -10.$**

35. **Using Lagrange's interpolation formula, express the function $(x^2 + 6x-1)/((x^2 -1)(x-4)(x-6))$ as a sum of partial fractions.**

36. **Find the error associated with Polynomial interpolation formula.**

37. **What do you mean by spline interpolation ? Obtain the (i) Linear (ii) quadratic and (iii) cubic spline interpolation polynomial that passes through the points (4, -43), (7, 83), (9, 327) and (12, 1053).**

38. **Using Lagrange's inverse interpolation formula, find the root of the equation f(x)=0 , given that f(30)= -30, f(34)= -13, f(38)= 3 and f(42)= -18.**

39. **Derive Newton divided interpolation formula and find the value of f(27) , given that f(20)= 0, f(34)= -13, f(35)= 3 and f(42)= -18.**

40. **The area A of a circle of diameter d is given for the following values:**

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

Calculate the area of a circle of diameter 105, using Newton backward interpolation formula.

41. **Find the number of men getting wages between Rs. 10 and 15 from the following data using Newton forward interpolation formula:**

Wages in rupees	0 - 10	10 - 20	20 - 30	30 - 40
frequency	09	30	35	42

42 . **The pressure p of wind corresponding to velocity v is given by the following data. Estimate p when v=25 by Newton backward interpolation formula.**

v	10	20	30	40
p	1.1	2	4.4	7.9

43. **The following table gives the viscosity of an oil as a function of temperature. Use Newton divided interpolation formula to find the viscosity of oil at a temperature of 140^0.**

Temp.^{0}	110	130	160	190
Viscosity	10.8	8.1	5.5	4.8

44. **Given the following values of x and y,find dy/dx and d^2y/dx^2 at (a) x=1.0, (b) x=1.15, (c) x=1.25 , (d) x=1.28.**

x	1.0	1.05	1.10	1.15	1.20	1.25	1.30
y	1.0000	1.0247	1.0488	1.0723	1.0954	1.1180	1.1401

45. **Using the following data, find x for which y is minimum and find this value of y.**

x	0.60	0.65	0.70	0.75
y	0.6221	0.6155	0.6138	0.6170

46. **Apply the method of Least squares to fit the curve $y=ax^2 + b/x$ to the following data:**

x	1	2	3	4
y	-1.51	0.99	3.88	7.66

47. **(a) Obtain the least square approximation of the form $f(x) = a\,x^b$ to the data**

x	0.5	0.6	0.7	0.8	1.0
f(x)	0.3136	0.4145	0.6146	0.8027	1.2542

(b) Obtain the Chebyshev polynomial approx. ofsecond degree to the function $f(x) = ax^3$ on [0,1].

(c) Define the orthogonality of a set of functions and explain the Gram-Schmidt orthogonalization process.

(d) Using Gram-Schmidt orthogonalization process, compute the first three orthogonal polynomials which are orthogonal on [0, 1] with respect to the weight function w(x)=1. Using these polynomials obtain the least square approximation of second degree for $f(x) = x^3 - 4x$ on [0, 1].

(e) Compute the first three polynomials which are orthogonal on [-1, 1] with respect to the weight function w(x)= $1/\sqrt{x^2+1}$. Using these polynomials obtain the least square approximation of second degree for $f(x) = x$ on [-1, 1].

48. **The voltage v across a capacitor at time t seconds is given by the following table:**

t	0	2	4	6	8
v	150	63	28	12	5.6

Use the method of Least squares to fit the curve of the form v = a e^t to this data.

49. **The following table gives the results of the measurements of train resistance ; V is the velocity in miles per hour, R is the resistance in pounds per ton:**

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

If R is related to V by the relation R = a + bV +Cv^2, find a, b and c.

50. **Evaluate** $\int_0^1 \frac{1}{1+x}dx$ **dx taking seven ordinates by applying Simpson 3/8 rule. Deduce the value of $\log_e 2$.**

51. **Evaluate the integrals (i)** $\int_0^{\pi/2} \sqrt{\sin x}\,dx$ **and (ii)** $\int_0^{\pi/2} \frac{x}{\sin x}dx$ **using (i) Simpson 1/3- rule and (ii) Trapezoidal rule of integration by taking eleven ordinates.**

52. **The velocity v of a particle at distance s from a point on its path is given by the table:**

s ft.	0	10	20	30	40	50	60
v ft./sec.	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft by using Simpson 1/3- rule. Compare the result with Simpson 3/8 rule.

53. **The following table gives the velocity v of a particle at time t :**

t (seconds)	0	2	4	6	8	10	12
v(m/sec.)	4	6	16	34	60	94	136

Find the distance moved by the particle in 12 seconds and also the acceleration at t=2 seconds.

54. **Evaluate the following integrals using using (i) Simpson 1/3- rule (ii) Simpson 3/8- rule and (iii) Trapezoidal rule of integration by taking seven ordinates respectively and find out which one is better.**

(i) $\int_2^4 (1+x^4)\,dx$ **(ii)** $\int_{-2}^2 e^{-x^2}dx.$

55. **Using Taylor series method, compute y(0.1) and y(0.2) to three places of decimal from the following:**

(i) dy/dx = 1 – 2xy given that y(0)=0 , (ii) dy/dx = xy +1, y(0)=1 (iii) dy/dx = x^2 + y^2 ,y(0)=1

56. **Solve the following initial value problem dy/dx = -xy^2 , y=2 at x=0 by Euler's modified method to obtain y at x=0.2 in steps of 0.1, correct to four decimal places.**

57. **Solve the differential equation dy/dx =2 +√(xy), where y(1)=1 by Euler's modified method to obtain y at x=2 in steps of 0.2, correct to four decimal places.**

58. **Apply Runge-Kutta method of order Two and Four respectively to solve the following initial value problems for y(0.2) by taking h=0.1 and hence compare the result.**
(i) 10 (dy/dx) = x^2 +y^2 , y(0)=1 **(ii) dy/dx =3x + y/2, y(0)=1**
(iii) dy/dx =(2xy +e^x)/(x^2+xe^x), y(0)=1

59. **Discuss the stability of Numerical methods for the solution of initial value problems.**

60. **Find the value of** $y(0.2)$ **and** $y(0.4)$, **using Runge-Kutta method of fourth order with** $h=0.2$ **given that** $\frac{dy}{dx}=\sqrt{x^2+y};\ \ y(0)=0.8.$

61. **Given that** $\frac{dy}{dx}=\frac{1}{x^2}-\frac{y}{x},\ \ y(1)=1, y(1.1)=0.996, y(1.2)=0.986$ **and** $y(1.3)=0.972,$ **find the values of** $y(1.4)$ **and** $y(1.5)$ **using Milne's Predictor-corrector method.**

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