Problems on Double and Triple Integrals

a) Problems on integrals when the limits are not provided

1. Evaluate $\iint_A \sqrt{xy(1-x-y)}\, dxdy$ Where A is the area bounded by x=0,y=0,and x+y= 1,

Ans. $2\pi/105$

2. Evaluate $\iint_A \sqrt{x^{m-1} y^{n-1}}\, dxdy$ Where A is the area bounded by x=0,y=0,and x+y= h

Ans. $\frac{h^{m+n}}{(m+n)} \frac{\Gamma m\, \Gamma n}{\Gamma m+n}$

3. Evaluate $\iint_A y\, dx\, dy$ over the area bounded by $y = x^2$ and x+y= 2

Ans. 36/5

4. Evaluate $\iint_A xy\, dx\, dy$ Where R is the area bounded by $x^2 = y, y^2 = -x$

Ans. -1/12

5. Evaluate $\iint_R xy / \sqrt{(1-y^2)}\, dxdy$ over positive quadrant of $x^2 + y^2 = 1$

Ans. 1/6

6 Evaluate $\int\int_A \sqrt{xy(1-x-y)}\, dxdy$ Where A is the area bounded by x=0,y=0,and x+y= 1

Ans $2\pi/105$

7. Evaluate $\iint_A x^p y^q\, dxdy$ over positive quadrant of $x^2 + y^2 = a^2$;

Ans $a^{p+q+2} / 2\,(p+q+2) \frac{\Gamma p+q/2.\ \Gamma q+1/2}{\Gamma p+q+2\ /2}$

8. Evaluate $\iint_A xy\, dx\, dy$ Where R is the area bounded by $x/a + y/b = 1$, x = 0, y = 0.

Ans. $a^2 b^2 / 24$

9 2. Evaluate $\iint_A \sqrt{x^{m-1} y^{n-1}}\, dxdy$ Where A is the area bounded by x=0,y=0,and x+y= h

Ans. $\frac{h^{m+n}}{(m+n)} \frac{\Gamma m\, \Gamma n}{\Gamma m+n}$

10. Evaluate $\iint_R xy\,(x+y)\, dx\, dy$ over the region R bounded by $x^2 = y, y^2 = x$

Ans 3/28

Problems on change of order of integration:

1. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dx\,dy}{(1+e^y)\sqrt{(1-x^2-y^2)}}$ Ans. $\pi/2 \log \frac{2e}{(1+e)}$

2. Show that $\int_0^a \int_{y^2}^{y} \frac{y\,dx\,dy}{(a-x)\sqrt{(ax-y^2)}} = \pi a/2$

3 Show that $\int_0^1 \int_1^{\sqrt{2-y^2}} \frac{y\,dx\,dy}{(2-x^2)\sqrt{(1-x^2y^2)}} = 1-\pi/4$

4. Show that $\int_0^a \int_0^{y} \frac{x\,dx\,dy}{\sqrt{(a^2-x^2)}\,(a-y)(y-x)} = \pi a$

5. Evaluate $\int_0^{\pi/2} \int_0^{y} \cos 2y \sqrt{1-a^2\sin^2 x}\,\frac{dx\,dy}{}$ Ans $1/3a^2\,[(1-a^2)^{3/2}-1]$

6. Show that $\int_0^1 \int_x^{1/x} \frac{y\,dx\,dy}{(1+xy)^2\,(1+y^2)} = (\pi-1)/4$

7. Evaluate $\int_0^1 \int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}} \frac{dx\,dy}{(x^2-2x+y-3)^2}$ Ans. $2/3-1/2.\log 3$

8. Evaluate $\int_0^1 dx \int_1^{\infty} e^{-y}y^x \log y\,dy$ Ans. $1/e$

9. Show that $\int_0^a \int_0^{a-\sqrt{a^2-y^2}} \frac{xy\log(x+a)dx\,dy}{(x-a)^2} = a^2/8.(2\log a+1)$

10. Change the order of integration in double integral

$$\int_0^a \int_{\sqrt{(a^2-y^2)}}^{y+a} f(xy)\,dx\,dy$$

Problems on direct evaluation of double integration

1. Evaluate

$$\int_0^1 \int_0^y xy \, dx \, dy$$

2. Evaluate

$$\int_0^1 \int_0^{1-x} (x+y) \, dy \, dx$$

3. Evaluate

$$\int_0^1 \int_0^1 dx \, dy / (1+x^2)(1+y^2)$$

4. Evaluate

$$\int_0^1 \int_0^{\sqrt{1/2(1-y^2)}} dx \, dy / \sqrt{(1-x^2-y^2)}$$

5. Evaluate

$$\int_{-1}^1 \int_0^{1-x} x^{1/3} y^{-1/2} (1-x-y)^{1/2} \, dx \, dy$$

6. Show that

$$\int_0^\infty \int_0^\infty e^{-x^2(1+y^2)} x \, dx \, dy = \pi/4$$

7. Show that

$$\int_0^1 \int_0^{2-x} y \, dy \, dx = 16/15$$

8. Show that

$$\int_0^1 \int_{x^2}^x xy(x+y) \, dx \, dy = 3/56$$

9. Show that

$$\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2) \, dy \, dx = 3/35$$

10. Show that

$$\int_0^1 \int_0^y xye^{-x^2} \, dx \, dy = 1/4e$$