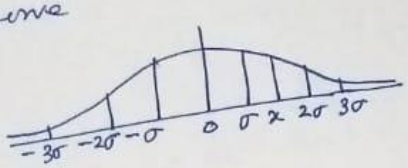
Normal distribution

1. Normal distribution is a continuous distribution
2. It can be derived as a limiting case of Binomial distribution as $n \to \infty$, $p \to \frac{1}{2}$
3. The median and mode coincide with the mean at $x = \mu$.
4. Normal curve

$f(x) \geq 0$



Total area under the curve

$\int_{-\infty}^{\infty} f(x)\,dx = 1$ $\quad \left[\int_{-\infty}^{0} f(x)\,dx = 0.5 \quad \int_{0}^{\infty} f(x)\,dx = 0.5\right]$

Probability density function

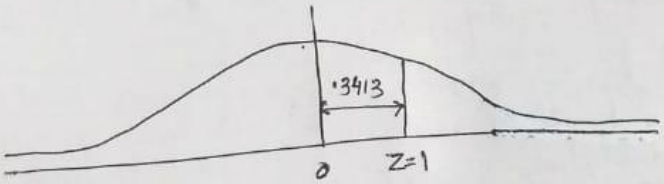
The Probability that an event will occur, as a function of some observed variable

5. The density function of Normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$, where μ = mean, σ = standard deviation.

6. If x = normal random variable then random variable $z = \frac{x-\mu}{\sigma}$

z	.00	.01	.09
0.0	.0000		
.1	.0398		
.2	.0793		
1.0	.3413		
3.0			



eg:1. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and S.d of 60 hours. Estimate the number of bulbs likely to burn for (a) more than 2150 hrs. (b) less than 1950 hrs. (c) more than 1920 hrs but less than 2160 hrs.

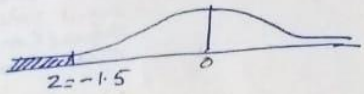
Soln

(a)

$$z = \frac{x-\mu}{\sigma} = \frac{2150-2040}{60} = 1.833$$

Area against $z = 1.833$ is $.4664$
Area right to the ordinate at $z = 1.833$ is $.5 - .4664 = .0336$
Number of bulb likely to burn more than 2150 hrs is $2000 \times .0336 = 67$ Ans.

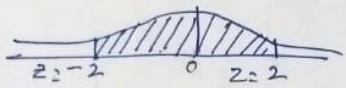
(b)



$$z = \frac{x-\mu}{\sigma} = \frac{1950-2040}{60} = -1.5$$

Area against $z = -1.5$ or 1.5 is $.4332$
Area left to the ordinate at $z = -1.5$ is $.5 - .4332 = .0668$
Number of bulb likely to burn less than 1950 hrs is $2000 \times .0668 = 134$

(c)



$$z_1 = \frac{x_1-\mu}{\sigma} = \frac{1920-2040}{60} = -2$$

$$z_2 = \frac{x_2-\mu}{\sigma} = \frac{2160-2040}{60} = 2$$

Area against $z = 2$ is $.4772$
Area between $z = -2$ & $z = 2$ is $.4772 + .4772 = .9544$
Number of bulb likely to burn more than 1920 hrs but less than 2160 hrs is $2000 \times .9544 = 1909$ Ans.

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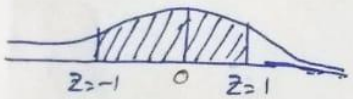
eg2 A manufacturer knows from experience that the resistance of resistors he produces is normal with mean $\mu = 100$ ohms and standard deviation $\sigma = 2$ ohm. What percentage of resistors will have resistance between 98 ohm & 102 ohm?

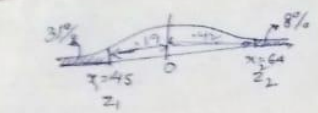


$$z_1 = \frac{x_1-\mu}{\sigma} = \frac{98-100}{2} = -1$$

$$z_2 = \frac{x_2-\mu}{\sigma} = \frac{102-100}{2} = 1.$$

Area against $z = 1$ is $.3413$.
Area between $z = -1$ & $z = 1$ is $.3413 + .3413 = .6826$
Percentage of resistors will have resistance between 98 ohm & 102 ohm is 68.26%. Ans.

Q.13 In a normal distribution 31% of the items are under 45 and 8% are over 64. Find Mean and Standard deviation of the distribution



$z_1 = \frac{x_1-\mu}{\sigma} = \frac{45-\mu}{\sigma}$ ①

$z_2 = \frac{x_2-\mu}{\sigma} = \frac{64-\mu}{\sigma}$ ②

$\int_{-\infty}^{z_1} f(z)dz = .31$ then $.5-.31 = .19$

$\therefore z_1 = -.5$

$\int_{z_2}^{\infty} f(z)dz = .08$ then $.5-.08 = .42$

$\therefore z_2 = 1.42$

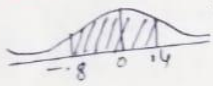
$\therefore$ from ① & ② $\frac{45-\mu}{\sigma} = -.5 \Rightarrow 45-\mu = -.5\sigma \Rightarrow \mu - .5\sigma = 45$

$\frac{64-\mu}{\sigma} = 1.42 \Rightarrow 64-\mu = 1.42\sigma \Rightarrow \mu + 1.42\sigma = 64$

$\sigma = 10$ & $\mu = 50$

Q.14 In a sample of 1000 cases, the mean of a certain test is 14 and S.d is 2.5. Assuming the distribution to be normal, find (i) how many students score between 12 and 15? (ii) how many score above 18? (iii) how many score below 8? (iv) how many score 16?

(i)



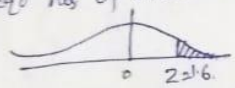
$z_1 = \frac{x_1-\mu}{\sigma} = \frac{12-14}{2.5} = -.8$, Area against $-.8$ is $.2881$

$z_2 = \frac{x_2-\mu}{\sigma} = \frac{15-14}{2.5} = .4$, Area again $.4$ is $.1554$

Area between $-.8$ & $.4$ is $.2881+.1554 = .4435$

Reqd nos of students $= 1000 \times .4435 = 4435$ i.e 444 Ans.

(ii)

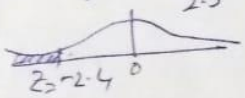


$z = \frac{x-\mu}{\sigma} = \frac{18-14}{2.5} = 1.6$, Area against $z = 1.6$ is $.4452$

Area above $z = 1.6$ is $.5 - .4452 = .0548$.

Reqd nos of students $= 1000 \times .0548 = 55$ Ans.

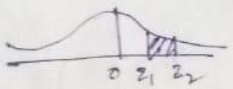
(iii) $z = \frac{x-\mu}{\sigma} = \frac{8-14}{2.5} = -2.4$, Area against $z = -2.4$ is $.4918$



Area below $z = -2.4$ is $.5 - .4918 = .0082$

Reqd nos. of students $1000 \times .0082 = 8$ Ans

(iv) $x_1 = 15.5$ & $x_2 = 16.5$



$z_1 = \frac{15.5-14}{2.5} = .6$, Area against $z_1 = .6$ is $.2257$

$z_2 = \frac{16.5-14}{2.5} = 1$ Area again $z_2 = 1$ is $.3413$

Area between z_1 & $z_2 = .3413 - .2257 = .1156$

Reqd nos. of students $= 1000 \times .1156 = 116$ Ans.

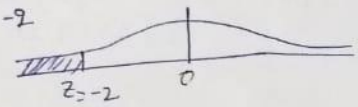
e.g.5 A sample of 100 dry battery cells tested to find the length of life produced the following results, $\mu = 12$ hrs & $\sigma = 3$ hrs. Assuming the data to be normally distributed, What percentage of battery cells are expected to have life (i) more than 15 hrs (ii) less than 6 hrs & (iii) between 10 & 14 hrs.

(i) $z_1 = \frac{x-\mu}{\sigma} = \frac{15-12}{3}$

$z = 1$

Area against $z=1$ is .3413.
Area right to the ordinate at $z=1$ is $.5 - .3413 = .1587$
The nos. of battery cells are expected to have life more than 15 hrs is 15.87%

(ii) $z = \frac{x-\mu}{\sigma} = \frac{6-12}{3} = -2$



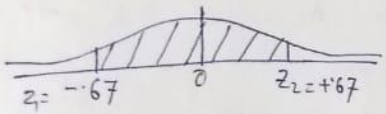
Area against $z=-2$ is .4772
Area left to the ordinate at $z=-2$ is $.5 - .4772 = .0228$
The nos of battery cells are expected to have life less than 6 hrs is 2.28%

(iii) $z_1 = \frac{x_1-\mu}{\sigma} = \frac{10-12}{3} = -.67$, Area against $z_1 = -.67$ is .2486

$z_2 = \frac{x_2-\mu}{\sigma} = \frac{14-12}{3} = +.67$. Area against $z_2 = .67$ is .2486.

Area between z_1 to z_2 is
$.2486 + .2486 = .4972$

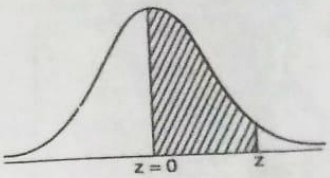
Reqd percentage is 49.72%



APPENDIX—I

Table 1 : NORMAL TABLE
AREAS UNDER THE STANDARD NORMAL

$$\text{CURVE} = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{z^2}{2}} dz$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2485	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.8729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4255	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4930	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4999	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993