

Lagrange's Interpolation (For unequal intervals)

Let $x: x_0 \quad x_1 \quad x_2 \quad x_3 \dots x_n$
 $y: y_0 \quad y_1 \quad y_2 \quad y_3 \dots y_n$

If the values of x are not at equal intervals

$$f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0$$

$$+ \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1$$

$$+ \dots + \frac{(x-x_0)(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\dots(x_n-x_{n-1})} y_n \quad \text{(L-I)}$$

This is known as Lagrange's Interpolation formula for unequal intervals.

e.g. 1. Using Lagrange's interpolation formula, find the values of y when $x=10$ from the following table:

$x:$	5	6	9	11
$y:$	12	13	14	16

Soln: Here $x_0=5, x_1=6, x_2=9, x_3=11$
 $y_0=12, y_1=13, y_2=14, y_3=16$

\therefore the intervals are unequal we have to use Lagrange's interpolation formula (L-I) above

$$f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times 13$$

$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= \frac{4 \times 1 \times (-1)}{(-1)(-4)(-6)} \times 12 + \frac{5 \times 1 \times (-1)}{1 \times (-3)(-5)} \times 13 + \frac{5 \times 4 \times (-1)}{4 \times (+3)(-2)} \times 14 + \frac{5 \times 4 \times 1}{6 \times 5 \times 2} \times 16$$

$$= 2 - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = 14 \frac{2}{3} \text{ Ans.}$$

Ex 2. Using Lagrange's formula, find the form of the function $f(x)$ given that

$x:$	0	2	3	6
$f(x):$	659	705	729	804

Soln: Since the intervals are unequal we have to use Lagrange's interpolation formula (L-I)

Here $x_0=0, x_1=2, x_2=3, x_3=6$
 $y_0=659, y_1=705, y_2=729, y_3=804$

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$f(x) = \frac{(x-2)(x-3)(x-6)}{(0-2)(0-3)(0-6)} \times 659 + \frac{(x-0)(x-3)(x-6)}{(2-0)(2-3)(2-6)} \times 705$$

$$+ \frac{(x-0)(x-2)(x-6)}{(3-0)(3-2)(3-6)} \times 729 + \frac{(x-0)(x-2)(x-3)}{(6-0)(6-2)(6-3)} \times 804$$

$$f(x) = -\frac{659}{36} (x^3 - 11x^2 + 36x - 36) + \frac{705}{8} (x^3 - 9x^2 + 18x)$$

$$- 81(x^3 - 8x^2 + 12x) + \frac{67}{6} (x^3 - 5x^2 + 6x)$$

$$f(x) = \frac{1}{72} (x^3 + 29x^2 + 1602x + 47448) \quad \text{Ans}$$

Q.1. A curve passes through the point $(0, 18), (1, 10), (3, -18)$ and $(6, 90)$.
 Find the slope of the curve at $x=2$

2. Find the distance moved by a particle and its acceleration at the end of 4 seconds, if the time versus velocity data is as follow:

$t:$	0	1	3	4
$v:$	21	15	12	10