Hermitian and Skew Hermitian Matrix

Conjugate $x+iy$ & $x-iy$ are conjugate to each other.

$$A=\begin{bmatrix} a & x-iy \\ x+iy & b \end{bmatrix}, \quad \text{then } \bar{A}=\begin{bmatrix} a & x+iy \\ x-iy & b \end{bmatrix}$$ is the conjugate matrix of A

Hermitian Matrix: A square matrix A is said to be Hermitian matrix if $A'=\bar{A}$, the element of the leading diagonal of a Hermitian matrix are real.

Let
$$A=\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$$

$$A'=\begin{bmatrix} 3 & 7+4i & -2-5i \\ 7-4i & -2 & 3-i \\ -2+5i & 3+i & 4 \end{bmatrix}, \quad \bar{A}=\begin{bmatrix} 3 & 7+4i & -2-5i \\ 7-4i & -2 & 3-i \\ -2+5i & 3+i & 4 \end{bmatrix}$$

$A'=\bar{A} \Rightarrow A$ is Hermitian Matrix

Skew Hermitian Matrix: A square matrix A is said to be Skew Hermitian matrix if $A'=-\bar{A}$. The leading diagonal elements of a Skew Hermitian matrix are either zero or purely imaginary.

e.g.
$$A=\begin{bmatrix} 0 & -2-i \\ 2-i & 0 \end{bmatrix}, \quad A'=\begin{bmatrix} 0 & 2-i \\ -2-i & 0 \end{bmatrix}, \quad \bar{A}=\begin{bmatrix} 0 & -2+i \\ 2+i & 0 \end{bmatrix}$$

$$-\bar{A}=\begin{bmatrix} 0 & 2-i \\ -2-i & 0 \end{bmatrix}$$

$\therefore A'=-\bar{A} \Rightarrow A$ is Skew Hermitian Matrix

e.g.
$$A=\begin{bmatrix} -i & 3+4i \\ -3+4i & 0 \end{bmatrix}, \quad A'=\begin{bmatrix} -i & -3+4i \\ 3+4i & 0 \end{bmatrix}, \quad -\bar{A}=\begin{bmatrix} -i & -3+4i \\ 3+4i & 0 \end{bmatrix}$$

$A'=-\bar{A} \Rightarrow A$ is Skew Hermitian Matrix

Show that

$$\begin{bmatrix} a & b+ic \\ b-ic & d \end{bmatrix}, \quad \begin{bmatrix} 1 & 2-3i & 3+4i \\ 2+3i & 0 & 4-5i \\ 3-4i & 4+5i & 2 \end{bmatrix}$$ are Hermitian Matrix