Managing Uncertainty in Supply Chain: Safety Stock

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Maximum Inventory = \( Q + ss \)
Min Inventory = \( ss \)
Average inventory = \( \frac{Q}{2} + ss \)

Reorder point or Reorder level

When safety stock is not considered, \( ROL = D_L = \text{lead time} \times \text{Demand} = LD \)

If safety stock is provided, \( ROL = D_L + ss \)

If review time is required, \( ROL = D_L + ss + (\text{Demand} \times \text{review time})/2 \)
Why safety stock is required in Supply Chain

1. Variability in demand forecast: Forecasts are never accurate
2. Demand Forecast=Systematic Component+ Random Component
3. \[ \sigma_D = 1.25 \times \text{Mean Absolute Deviation} = 1.25 \times \frac{1}{n} \sum_{t=1}^{n} |F_t - D_t| \]
4. Uncertain demand and production less than the planned may cause inventory stock out.
5. During the stock out period, lost sale and back orders may appear which is a kind of loss to the company.
6. Therefore, an amount of additional inventory is kept to counter the uncertain demand during lead time, called as safety inventory or safety stock.
7. Safety stock facilitates good product availability and customer satisfaction.
8. Safety stock causes additional cost to the firm. (cost of capital, holding cost etc.)
9. The uncertainty in Demand and lead time affect the safety inventory.
10. The desired cycle service level also affects the safety inventory.
Problems to be addressed

1. What appropriate level of safety inventory the firm should carry?

2. What actions can be taken to improve product availability while reducing safety inventory?
Why the demand is uncertain?

- Quality of the product of competitors
- Price of the product of competitors
- Affordability of the customers
- Upcoming Product
- Availability of the Product in Market
- Variation of Lead time
- Natural calamity/pandemics etc.
- Seasonal Effect
Determining optimum level of safety inventory
Measuring Demand Uncertainty: coefficient of correlation

1. Consider that the demand for each period \(i=1,2,3\ldots L\) is normally distributed with mean \(D_i\) and standard deviation \(\sigma_i\)

2. The total demand for all \(L\) periods is also normally distributed with a mean \(P\) and standard deviation \(\Omega\) then

\[
P = \sum_{i=1}^{L} D_i \quad \Omega = \sqrt{\sum_{i=1}^{L} \sigma_i^2 + 2 \sum_{i>j} \rho_{ij} \sigma_i \sigma_j}
\]

Where \(\rho_{ij}\) is the coefficient of correlation of periods \(i\) and \(j\)

If \(\rho_{ij} = 1\) Demand of two periods is positively correlated

If \(\rho_{ij} = -1\) Demand of two periods is negatively correlated

If \(\rho_{ij} = 0\) Demand of two periods is independent

Assuming that the demand during each \(L\) periods is independent and normally distributed with a mean of \(D\) and standard deviation of \(\sigma_D\)

Therefore, the total demand during \(L\) periods is normally distributed with a mean \(D_L\) and standard deviation \(\sigma_L\)

\[
D_L = LD \quad \text{and} \quad \sigma_L = \sigma_D \sqrt{L}
\]
Measuring Demand Uncertainty: \textit{coefficient of variation (cv)}

Ratio of standard deviation to the mean

- \( cv = \frac{\sigma}{\mu} \)
- \( cv \) represents the size of uncertainty relative to the demand
- Case 1 (Fig. 1): \( \sigma = \mu = 3 \), \( \frac{\sigma}{\mu} = 1 \) (High variance)
- Case 2 (Fig. 2): \( \sigma = 1 \), \( \mu = 3 \), \( \frac{\sigma}{\mu} = \frac{1}{3} \) (Low Variance)
Measures of Product Availability: Fill Rate & CSL

• Fill Rate ($f_r$):
  • Product Fill Rate: The probability of event that the product demand is satisfied from available inventory (fraction of product demand satisfied from available inventory)
  • Order Fill Rate: The probability of the event that all orders can be satisfied from available inventory (fraction of complete orders satisfied from available inventory).
  • Product fill rate > Order fill rate

• Cycle Service Level (CSL): Fraction of replenishment cycle that ends with customer demand being met. Or The probability of not having any stockout in a replenishment cycle.

• CSL = Prob(Demand during lead time of L weeks ≤ ROP)
• Demand during lead time $L = C = LD$
• Standard deviation of demand $\sigma_L = \sigma_D \sqrt{L}$
• CSL = $F(ROP, D_L, \sigma_L)$
Measures of Product Availability: Expected Shortage per replenishment cycle (ESC)

- **Expected Shortage per replenishment cycle (ESC):** Average units of demand that are not satisfied from the available inventory in a replenishment cycle.

\[ ESC = \int_{x=ROP}^{\infty} (x - ROP)f(x)dx \]

Where \( f(x) \) is the density function of demand distribution during lead time

- When demand is normally distributed,

\[
ESC = -ss \left[ 1 - F_S \left( \frac{ss}{\sigma_L} \right) \right] + \sigma_L f_S \left( \frac{ss}{\sigma_L} \right)
\]

Where, \( F_S \) is standard normal cumulative distribution function, \( f_S \) is standard normal density function and \( ss \) is safety stock

\[
F_S = z = \frac{x - D_L}{\sigma_L}
\]

\[
f_S(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}
\]

Safety stock (ss)=ROP-D_L

- Fill rate \( (f_r) = 1 - \frac{ESC}{Lot \ size} \)
# Uncertainty in demand and/or lead time

- **Uncertain demand and constant lead time**
  - \( D_L = LD \)
  - \( \sigma_L = \sigma_D \sqrt{L} \)
  - \( SS = z\sigma_L \)

- **Uncertain demand and uncertain lead time**
  - \( D_L = LD \)
  - \( \sigma_L = \sqrt{L\sigma_D^2 + D^2S_L^2} \)
  - \( SS = z\sigma_L \)

Where,

- \( \sigma_D \) = standard deviation of demand per period
- \( D \) = mean of demand per period
- \( \sigma_L \) = standard deviation of lead time
- \( S_L \) = standard deviation of lead time of replenishment
- \( z \) = CSL
- \( L \) = mean of lead time of replenishment
Problem 1

Weekly demand of a specific type of laser printer in a warehouse is normally distributed with mean 300 and standard deviation of 48 units. The replenishment lead time is 3 weeks. The warehouse owner decides to achieve a CSL of 85% and review the inventory continuously. Evaluate the safety inventory and reorder point. If the reorder quantity is 1200 printers, calculate the fill rate.

**Solution**  
CSL = 85% or 0.85

We have to compute \( z \) corresponding to 0.85 CSL

<table>
<thead>
<tr>
<th>CSL</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>0.8485</td>
<td>0.85</td>
<td>0.8508</td>
</tr>
</tbody>
</table>

Running the linear interpolation

\[
y_2 = \frac{(x_2 - x_1)(y_3 - y_1)}{x_3 - x_1} + y_1
\]

\[
= \frac{(0.85 - 0.8485)(1.04 - 1.03)}{0.8508 - 0.8485} + 1.03 = 1.0365 = z
\]

\[
D_L = LD = 300 \times 3 = 900
\]

\[
\sigma_L = \sigma_D \sqrt{L} = 48 \sqrt{3} = 83.14
\]

If the reorder quantity is 1200 printers, calculate the fill rate.
Safety stock \( SS = z \sigma_L = 1.0365 \times 83.14 = 86.175 \)  
Reorder point = \( \Delta S + D_L = 986.17 \)  
Lot size is given \( \Theta_i = 1200 \) units  
\[ f_r = 1 - \frac{ESC}{\Theta_i} \]  
\[ ESC = -\Delta \left[ 1 - F_\beta \left( \frac{\Delta}{\sigma_L} \right) \right] + \sigma_L f_\beta \left( \frac{\Delta}{\sigma_L} \right) \]  
\[ = -\Delta \left[ 1 - F_\beta (Z) \right] + \sigma_L f_\beta (Z) \]  
Here \( F_\beta (1.0365) = 0.85 \) , \( \sigma_L f_\beta (1.0365) \) \( f_\beta (1.0365) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} = 0.233 \)  
\[ ESC = -86.175 [1-0.85] + 83.14 \times 0.233 \]  
\[ = 12.93 + 19.37 = 6.44 \]  
Fill rate \( = f_r = 1 - \frac{ESC}{\Theta_i} = 1 - \frac{6.44}{1200} = 0.9946 \)  
or \( f_r = 99.46\% \)
Problem 2 Self practice

Weekly demand and replenishment lead time of a specific type of laser printer in a warehouse are normally distributed. The mean and standard deviation of demand and lead time are 3000 printers, 8 days and 800 printers, 7 days respectively. The warehouse owner decides to achieve a CSL of 75% and reviews the inventory continuously. Evaluate the safety inventory and reorder point. If the reorder quantity is 5500 printers, calculate the fill rate.
Product Substitution: Use of one product to satisfy the demand of a different product

- **Manufacturer driven one way substitution**
  - The manufacturer substitutes a higher valued product for a lower valued product in case of high demand or shortages
  - The manufacturer has two choices
    - Delay or deny the customer order
    - Substitute the higher valued product at same price or with a minimal price difference.
  - The manufacturer must aggregate the safety stock of all concerned products

- **Customer driven two way substitution**
  - The customer makes the decision to substitute the product
  - Example: grocery items
  - Aggregation of inventory works
  - The shop manager should recognize and encourages the customer to go for substitution
Component Commonality
- Standardization of components across the wide product range
- Reduces the overall inventory throughout the supply chain

Postponement
- Ability of supply chain to delay product differentiation or customization until closer to the time the product is sold
- Reduces the overall inventory throughout the supply chain

Example
New gas connection