Machine Tool Design:

Regulation of speed and feed rates

Need to regulate cutting speed and feed rates.

The machining cost, \( C = C_{mt} + C_{npt} + C_{tc} + C_t \)

Where,

\( C_{mt} = (W+E) t_m \) represents the cost of machining time; \( W \) is wage rate, \( E \) is cost of operating the machine tool per unit time and \( t_m \) is machining time.

\( C_{npt} = (W+E) t_{npt} \) represents the cost of nonproductive time; \( t_{npt} \) is nonproductive time such as loading and unloading, idle travel of cutting tool etc.

\( C_{tc} = \frac{(W+E) t_c}{Q} \) represents tool changing cost per component;

\( t_c \) is tool changing time, \( Q \) is no of components machined during the tool life.

\( C_t = \frac{T_c}{Q} \) represents the cost of tool per component

\( T_c \) is cost of a tool for a period equal to tool life.

\( = \) Tool cost / no. of regrindings

Generalized tool life equation, \( T = \frac{wa^{1/c}}{machining\ cost} \)

Where \( W = \) Cutting speed = \( \frac{11 dn}{1000} \) m/min

\( a = \) feed rate

\( c = depth\ of\ cut \)

Since \( n = \frac{1000 V}{\pi d} \), thus we need to have a stepless regulation of speed for any diameter of the workpiece. However, due to the principle of drive motors, a constant torque cannot be produced at all speed levels. Also, regulating the speed precisely at some specific value is difficult. Hence, stepless gearbox is needed.
\[ \sqrt{T^n} = C \]

Cost per piece

\[ N = \frac{\pi d n}{1000} \]

Economical cutting speed \rightarrow Cutting speed

\[ n_1 = \frac{1000v_{opt}}{\pi d_1} \]

\[ n_2 = \frac{900v_{opt}}{\pi d_2} \]
Selection of range of spindle speeds

The selection of spindle speed depends on the following factors:

1. Workpiece material properties,
   - Hardness, machinability
2. Form stability and wear resistance of tool material
3. Shape of cutting tool
4. Operation requirement
   - Finishing operation requires high spindle speed
   - Drilling and screwing require low and medium spindle speed
5. Process capability of machine
   - Capability of machine to achieve close tolerances
MACHINE TOOL DRIVES

Source of power:
- Individual drive from its motor
- Group drive from line shaft

Range of speed:
- Stepped drive
- Steppless drive

Transmission system:
- Mechanical
  - Belt driven
  - Gear driven
- Electrical
- Hydraulic
- Pneumatic

Nature of motion:
- Drives for producing rotational movement
- Drives for producing rectilinear movement
- Slider-crank mechanism
- Crank- rocker mechanism
- Screw-nut mechanism
- Rack-pinion mechanism
- Cam mechanism

Single stage transmission:
- Stepped drive

Stepless drive
Laws of Stepped Regulation of Speed

1. Arithmetic progression
   \[ \eta_1 = n \]
   \[ \eta_2 = \eta_1 + a = n + a \]
   \[ \eta_3 = \eta_2 + 2a \]
   \[ \eta_4 = \eta_3 + 3a \]
   \[ \eta_5 + \ldots \]
   \[ \eta_z = \eta_1 + (z-1)a \]
   \( z = \) number of steps
   \( a = \) common difference of arithmetic progression

Let upper and lower limits of spindle rpm are \( \eta_{nx} \) and \( \eta_{nx+1} \) respectively. Therefore,

the upper range of \( \text{rpm} \) dia = \( dx = \frac{1000 V}{\pi \eta_{nx}} \)

lower range of \( \text{rpm} \) dia = \( dx+1 = \frac{1000 V}{\pi \eta_{nx+1}} \)

Hence the diameter range served by the spindle

\[ \Delta dx = \frac{1000 V}{\pi} \left( \frac{1}{\eta_{nx}} - \frac{1}{\eta_{nx+1}} \right) \]

2. Geometric progression
   \[ \eta_1 = n, \quad \eta_2 = n_1 \phi, \quad \eta_3 = n_1 \phi^2, \quad \ldots \quad \eta_z = n_1 \phi^{z-1} \]
   \( \phi = \) geometric progression ratio, \( z = \) no of steps

\[ \phi = \left( \frac{n_2}{n_1} \right)^{1 \over z-1} \]

\[ \Delta dx = \frac{1000 V}{\pi} \left( \frac{1}{\eta_{nx}} - \frac{1}{\eta_{nx+1}} \right) \]
ARITHMETIC PROGRESSION

Problem: 1. Estimate the diameters range for different rpm values between 50 rpm and 440 rpm with 14 speed steps.

Consider the cutting speed \( v = 25 \text{ m/min} \)

Solution: \( n_1 = 50 \text{ rpm}, n_2 = 440 \text{ rpm}, v = 25 \text{ m/min}, z = 14 \)

\[ n_2 = n_1 + (z-1)a \Rightarrow 440 = 50 + 13a \Rightarrow a = 30 \]

<table>
<thead>
<tr>
<th>( n_x ) (rpm)</th>
<th>( dx ) (mm)</th>
<th>( \Delta dx ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>159.15</td>
<td>59.68</td>
</tr>
<tr>
<td>80</td>
<td>99.47</td>
<td>27.13</td>
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<td>110</td>
<td>72.34</td>
<td>15.50</td>
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<td>140</td>
<td>56.84</td>
<td>10.03</td>
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<td>170</td>
<td>46.81</td>
<td>7.02</td>
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<td>200</td>
<td>39.79</td>
<td>5.19</td>
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<td>230</td>
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<td>3.99</td>
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<td>260</td>
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<td>290</td>
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<td>320</td>
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<tr>
<td>350</td>
<td>22.74</td>
<td>1.79</td>
</tr>
<tr>
<td>380</td>
<td>20.94</td>
<td>1.53</td>
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<tr>
<td>410</td>
<td>19.41</td>
<td>1.32</td>
</tr>
<tr>
<td>440</td>
<td>18.09</td>
<td></td>
</tr>
</tbody>
</table>

Conclusion:

(i) In low rpm range the \( \Delta dx \) is larger, therefore more steps are required to be added.

(ii) In high rpm range the \( \Delta dx \) is very small between the steps, therefore some values of speed (or steps) are redundant and should be removed.
Problem 2: Estimate the diameter range for different rpm ranges between 50 rpm to 440 rpm with 14 speed steps. Consider the cutting speed \( v = 25 \text{ m/min} \).

<table>
<thead>
<tr>
<th>( \eta x (\text{rpm}) )</th>
<th>( dx (\text{mm}) )</th>
<th>( \Delta dx (\text{mm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.0</td>
<td>159.15</td>
<td>24.52</td>
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<tr>
<td>59.1</td>
<td>134.63</td>
<td>20.74</td>
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<tr>
<td>69.9</td>
<td>113.89</td>
<td>17.54</td>
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<tr>
<td>82.6</td>
<td>96.35</td>
<td>14.84</td>
</tr>
<tr>
<td>97.6</td>
<td>81.51</td>
<td>12.56</td>
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<tr>
<td>115.4</td>
<td>68.95</td>
<td>10.62</td>
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<tr>
<td>136.4</td>
<td>58.33</td>
<td>8.99</td>
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<tr>
<td>161.3</td>
<td>49.34</td>
<td>7.60</td>
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<td>190.6</td>
<td>41.74</td>
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<td>225.3</td>
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<td>5.44</td>
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<td>266.4</td>
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<td>314.9</td>
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<td>372.2</td>
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<td>3.29</td>
</tr>
<tr>
<td>440.0</td>
<td>18.09</td>
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</tr>
</tbody>
</table>

**Conclusion:**

(i) The low rpm values can be brought closer.

(ii) The high rpm values can be widened.
Harmonic Progression - Let the spindle speed \( n_1, n_2, \ldots, n_{x+1}, n_x \) be in harmonic progression.

\[
\Delta n_x = dx - dx+1 = \frac{1600\pi}{11} \left( \frac{1}{n_x} - \frac{1}{n_{x+1}} \right) = \text{Constant}
\]

\[
\frac{1}{n_x} - \frac{1}{n_{x+1}} = c
\]

\[
m_{x+1} - n_x = c \cdot n_x \cdot n_{x+1}
\]

\[
m_{x+1} \left( 1 - c \cdot n_x \right) = n_x
\]

\[
m_{x+1} = \frac{n_x}{1 - c \cdot n_x}
\]

\[
n_1 = n
\]

\[
n_2 = \frac{n_1}{1 - c \cdot n_1}
\]

\[
n_3 = \frac{n_2}{1 - c \cdot n_2}
\]

\[
n_4 = \frac{n_1}{1 - 2c \cdot n_1}
\]

\[
n_5 = \frac{n_1}{1 - 3c \cdot n_1}
\]

\[
n_2 = \frac{n_1}{1 - (3-1) \cdot c \cdot n_1}
\]

Note: In harmonic progression, the rpm range values in high range are too wide apart making this range uneconomical for exploitation.
Logarithmic progression: Let the spindle speed \( \eta_1, \eta_2, \ldots, \eta_3, \eta_4 \) constitute a logarithmic progression.

The value of \( \Delta dx \) in this progression

\[
\Delta dx = 2M dx^p
\]

\[
dx - dx + 1 = 2M dx^p
\]

\[
dx + 1 = dx - 2M dx^p
\]

\[
= dx \left[ 1 - \frac{2M}{dx^p} \right]
\]

\[
= \frac{1000V}{T_{inx}} \left[ 1 - \frac{2M}{\left( \frac{1000V}{T_{inx}} \right)^p} \right]
\]

\[
\frac{1000V}{T_{inx} + 1} = \frac{1000V}{T_{inx}} \left[ 1 - \frac{2M}{\left( \frac{T_{inx}}{1000V} \right)^{1-p}} \right]
\]

\[
\frac{dx}{dx + 1} = \left[ 1 - 2M \left( \frac{T_{inx}}{1000V} \right)^{1-p} \right] = 1 - \phi
\]

Assume

\[
\eta_1 = \eta_1 \phi_1, \quad \frac{1}{\phi_1} = 1 - 2M \left( \frac{T_{inx}}{1000V} \right)^{1-p}
\]

\[
\eta_2 = \eta_1 \phi_2, \quad \frac{1}{\phi_2} = 1 - 2M \left( \frac{T_{inx}}{1000V} \right)^{1-p}
\]

\[
\eta_3 = \eta_2 \phi_3, \quad \frac{1}{\phi_3} = 1 - 2M \left( \frac{T_{inx}}{1000V} \right)^{1-p}
\]

\[
\eta_4 = \eta_3 \phi_4, \quad \frac{1}{\phi_4} = 1 - 2M \left( \frac{T_{inx}}{1000V} \right)^{1-p}
\]
### HARMONIC PROGRESSION

<table>
<thead>
<tr>
<th>( n_x ) (rpm)</th>
<th>( d_x ) (mm)</th>
<th>( \Delta d_x ) (mm)</th>
</tr>
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<tbody>
<tr>
<td>50.0</td>
<td>159.15</td>
<td>15.00</td>
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<tr>
<td>55.2</td>
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<td>61.6</td>
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<td>80.3</td>
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<tr>
<td>869.7</td>
<td>9.15</td>
<td>15.00</td>
</tr>
</tbody>
</table>

**Conclusion**

* Good in low spindle speed range
* Poor in high spindle speed range as the rpm values in high speed range are too wide apart.