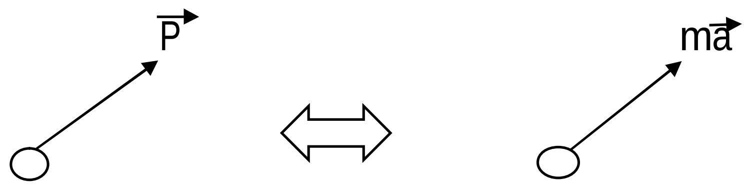
PRINCIPLE OF IMPULSE AND MOMENTUM

Consider a particle of mass 'm' acted upon by a force $\vec{P}$



From Newton's second law

$$\vec{P} = m\vec{a} = m\, d\vec{v}/dt$$

$$\vec{P} = d(m\vec{v})/dt$$

Define $m\vec{v}$ = instantaneous momentum of particle(body).

Separate variables

$$d(m\vec{v}) = \vec{P}\,dt$$

integrate between time t_1 and t_2 when velocity is $\vec{v}_1$ and $\vec{v}_2$ respectively

$$\int_{v1}^{v2} d(m\vec{v}) = \int_{t1}^{t2} \vec{P}\,dt$$

$$[m\vec{v}]_{\vec{v}1}^{\vec{v}2} = \int_{t1}^{t2} \vec{P}\,dt$$

$$m\vec{v}_2 = m\vec{v}_1 + \int_{t1}^{t2} \vec{P}\,dt$$

Define $\int_{t1}^{t2} \vec{P}\,dt$ = Impulse of force $\vec{P}$ during time t_1 to t_2.

$= \vec{I}_{1\rightarrow 2}$

$$m\vec{v}_2 = m\vec{v}_1 + \vec{I}_{1\rightarrow 2}$$

If more than one force is acting

$\vec{mv_2} = \vec{mv_1} + \sum \vec{I}_{1 \to 2}$

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system of bodies

$\sum \vec{mv_2} = \sum \vec{mv_1} + \sum \sum \vec{I}_{1 \to 2}$-----(external impulses)

Impulse of a force $\vec{P}$ during the interval t_1 to t_2.

General Case,

$\vec{I}_{1 \to 2} = \vec{i} \int_{t1}^{t2} P_x dt + \vec{j} \int_{t1}^{t2} P_y dt + \vec{k} \int_{t1}^{t2} P_z dt$

Note $\vec{P}$ = Function of time $\vec{P} = P_x\vec{i} + P_y\vec{j} + P_z\vec{k}$

Where $P_x = P_x(t)$

$P_y = P_y(t)$

$P_z = P_z(t)$

Special Case

1. If $\vec{P}$ is a CONSTANT VECTOR

   $\vec{I}_{1 \to 2} = \int_{t1}^{t2} \vec{P} dt = \vec{P} \int_{t1}^{t2} dt = \vec{P} [t]_{t1}^{t2}$

   $= \vec{P}(t_2 - t_1) = \vec{P}(\Delta t)$

2. If $\Delta t = t_2 - t_1$ is very small,
   Then $\vec{P}$ can be treated as constant.
   In such a situation $\vec{P}$ is called IMPULSIVE force.

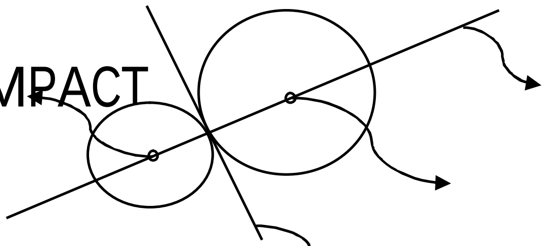
$$\vec{I}_{1\to 2} = P(\Delta t)$$

COLLISION OF TWO BODIES(IMPACT)

1. Definition- If geometric centre of the bodies are same, then called impact is CENTRAL otherwise ECCENTRIC.

C_1,G_1LINE OF IMPACT

C_2,G_2



COMMON NORMAL

In central impact, bodies are in translation.

In eccentric impact, bodies are in translation + rotation.

2. If impact is central then -

a) if velocities of both particles before or after impact are along line of impact,

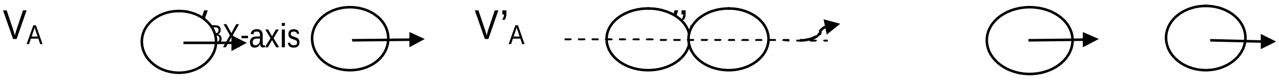
then the impact is called DIRECT IMPACT.

b) otherwise OBLOQUE IMPACT.

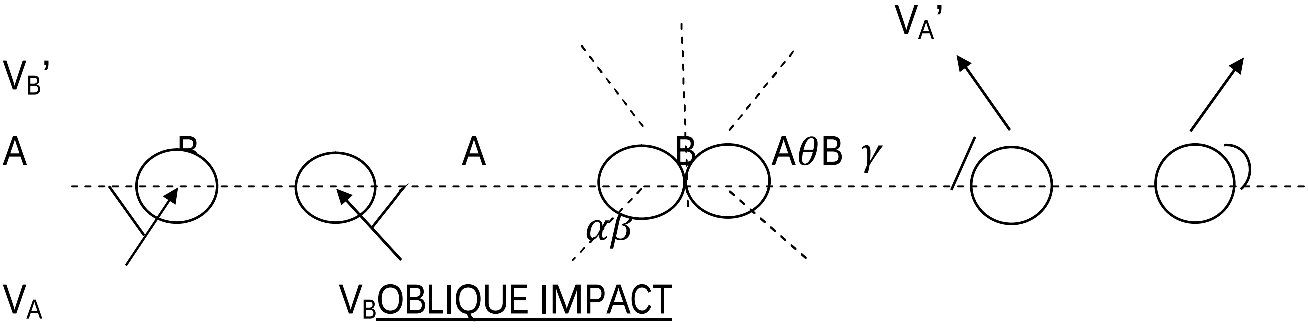
IMPACT

CENTRALECCENTRIC

DIRECT OBLIQUE



Before Impact DIRECT IMPACT After Impact



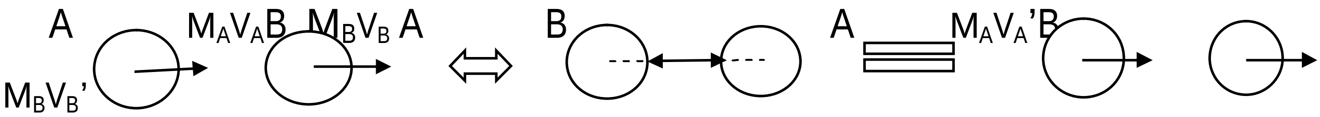
OBLIQUE IMPACT

CENTRAL , DIRECT IMPACT OF TWO BODIES

Given : Masses of bodies as M_A and M_B.

Velocities of bodies before impact V_A and V_B.

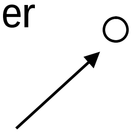
To Find : Velocities of both bodies after impact as V_A' and V_B'.



Before impact $\int Ndt$ After impact

For impact, $V_A > V_B$

Considering both bodies(particles) together



$(\sum mv)_{After\ Impact} = (\sum mv)_{Before\ Impact} + \sum\sum I_{1-2}$

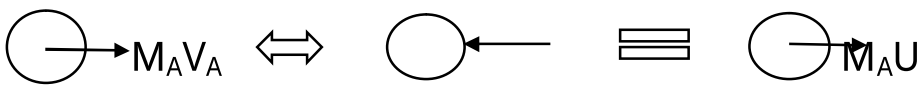
As the impulse received are internal hence the system momenta is conserved.

$$M_AV_A' + M_BV_B' = M_AV_A + M_BV_B \text{ ----------- } (1)$$

The second equation is obtained from Newton's Law of RESTITUTION

Study of body "A" during Deformation/Restitution

∫Pdt



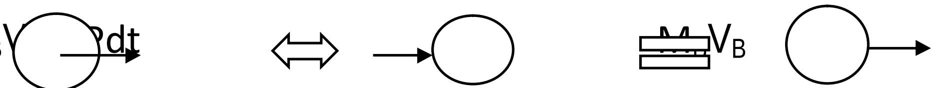
Before Impact During Impact At the end of Deformation

$M_AV_A - \int Pdt = M_AU$ or $\int Pdt = M_A(V_A-U)$ ------------- (2)

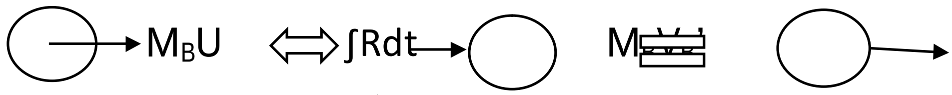
At the end of Deformation During Restitution After impact

$M_AU - \int Rdt = M_AV_A'$ ⟹ $= M_A(U-V_A')$ ----- (3) -------

Study of body "B" during Deformation/Restitution



$M_BV_B + \int Pdt = M_BU$ ⟹ $\int Pdt = M_B(U-V_B)$ ---------------------- (4)



$M_BV_B + \int Rdt = M_BV_B'$ ⟹ $\int Rdt = M(V_B'-U)$ ------------------ (5)

Define ,e = ∫ Rdt\∫ pdt = Coefficient of restitution

From equation 2 & 3, $e = u-v_a^1 \backslash v_a-u = v_b^1-u \backslash u-v_b$ from equation 2&5

Newton's Law of Restitution $e = v_b^1 - v_a^1 \backslash v_a - v_b$ or $v_b^1 - v_a^1 = e(v_a - v_b)$

(Relative Velocity of separation) =e (Relative vel. Of approach) before input

If, e =1 ➡impact is elastic

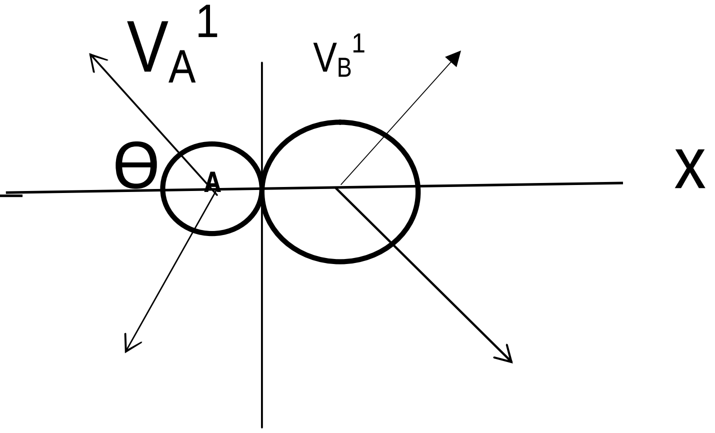
$\int Rdt = \int Pdt$

If ,e =0 ➡$\int Rdt = 0$

$v_a^1 = v_b^1 = u$

OBLIQUE IMPACT

Line of impact

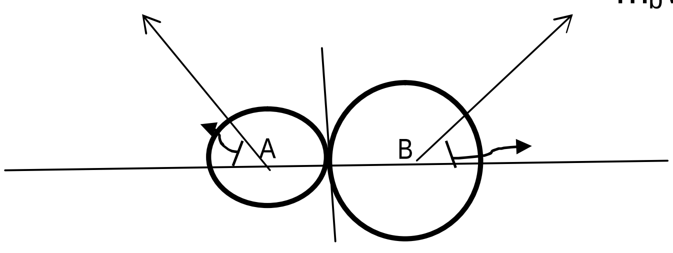


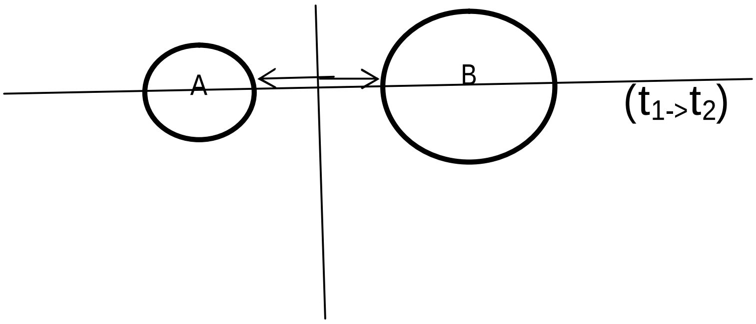
Before impact

$m_a v_a^1$

$m_b v_b^1$

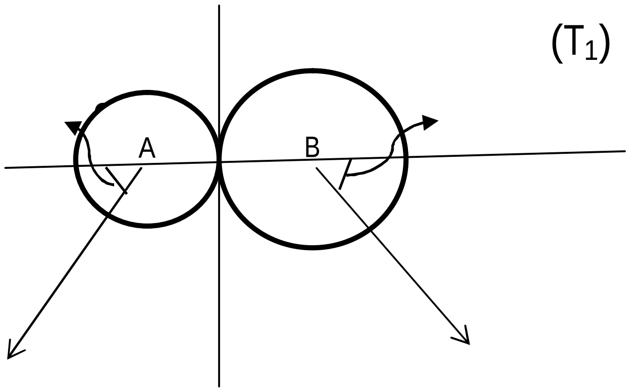
θv (t$_2$)





α

$m_A V_A m_B v_b$



$(\sum moments)_x = (\sum momenta)_x$

$m_A v_A \cos\alpha - m_B v_B \cos\beta = -m v_A^1 \cos\Theta = \ m_B v_B^1 \cos\gamma$----------(1)

□ +A ⟹ imp.{momentum eqn along y -dir^n}

$m_A v_A \sin\alpha = m_A v_A^1 \sin\Theta$ ------------(2)

□ +B ⟹ imp.{momentum eqn along y-dir^n

$m_B v_B \sin\beta = m_B v_B^1 \sin\gamma$ --------------(3)

Newton's Law of restitution is valid along x dir^n

$V_B^1 \cos\gamma = v_B^1 = \cos\Theta = e\ [v_A \cos\alpha + v_B \cos\beta\ ]$------------(4)