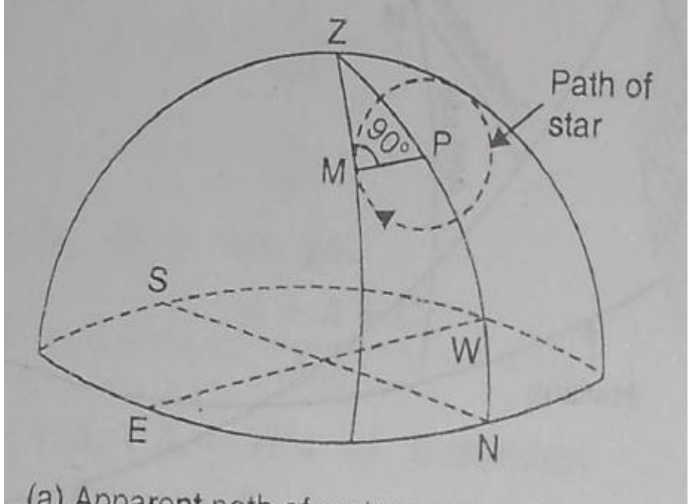
Field Astronomy
Circumpolar Star at Elongation

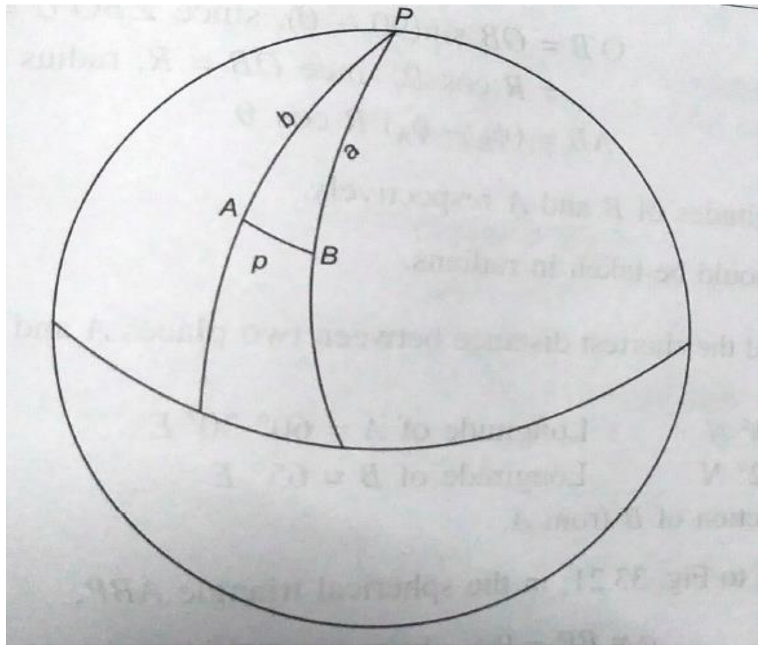
- At elongation a circumpolar star is at the farthest position from the pole either in the east or west.
- When the star is at elongation (East or West), which is perpendicular to the N-S line.
- Thus the ∠ ZMP is 90° as shown in the figure below.



(a) Apparent path of a star as seen from outside the celestial sphere

Distances between two points on the Earth's surface

➢ **Direct distance**



From the fig. above, in the spherical triangle APB

$\angle P$ = Difference between longitudes of A and B

BP = a = co-latitude of B = 90 – latitude of B

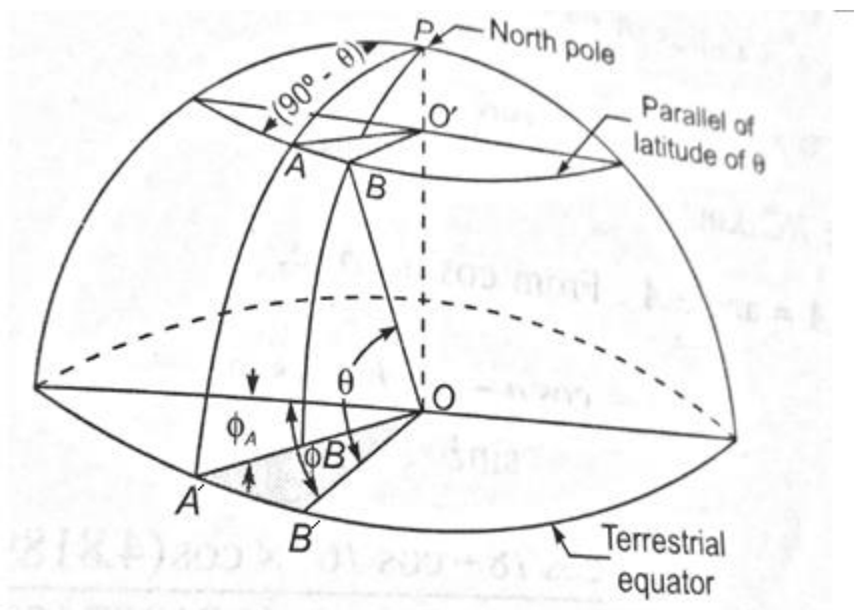
AP = b = co-latitude of A = 90 – latitude of A

Apply the cosine rule:

$$CosP = \frac{\cos p - \cos a \cos b}{\sin a \sin b}$$

Find the value of *p* = AB
Then the distance AB = arc length AB = '*p*' in radians × radius of the earth

➢ Distance between two points on a parallel of latitude



Let A and B be the two points on the parallel latitude θ. Let A' and B' be the corresponding points on the equator having the same longitudes (Φ_{B}, Φ_{A}).

Thus from the Δ O'AB

AB = O'B(Φ_B - Φ_A)

From the Δ O'BO

O'B = OB' sin (90- θ) Since $\angle BOO' = 90^o$

= R cos θ where R=Radius of the Earth.

AB= (Φ_B - Φ_A) R cos θ where Φ_B , Φ_A are longitudes of B and A in radians.

Practice Problems

1. Find the shortest distance between two places A and B on the earth for the data given below:
   Latitude of A = 14° N Longitude of A = 60°30'E
   Latitude of B = 12° N Longitude of A = 65° E
   Find also direction of B from A.
2. Calculate the distance in kilometers between points A and B along the parallel of latitude, for the following data:
   Longitude of A = 36°30'W; Longitude of B = 45°30'W; Latitude of A and B = 30° N

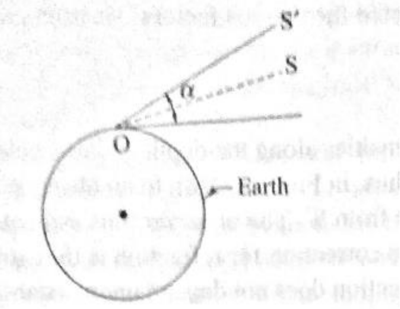
Corrections to Astronomical Observations

➢ Correction for refraction

- Refraction of light rays due to varying air densities along the depth makes a celestial body appear higher than it actually is.
- The correction for refraction is thus subtracted from the observed altitude and it is independent of the distance.
- It can be determined from refraction tables or may be calculated by the following formulae:

  Correction for refraction in altitude α = 58'' cot α

  Correction for refraction in zenith distance z = 58'' cot z



(a) Correction for refraction

➢ Correction for parallax

- This correction is made in the case of observations made to the Sun only.
- To other stellar bodies the correction is very small as the bodies are far away from the Earth.
- Refer the fig. (b) and ΔO'OS

  OO'/sin OSO' = O'S/ sin O'OS

  or (OO'/ O'S) sin O'OS = sin OSO'

  sin O'OS = sin (90+α) = cos α

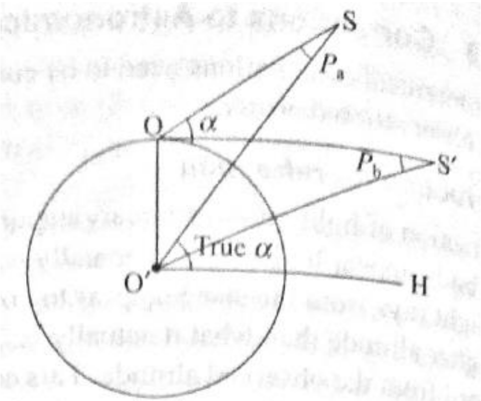
  or sin P_a = (OO'/ O'S) cos α (Since $\angle$ OSO' = P_a)

  Also (OO'/ O'S) = (OO'/ O'S') = sin O'SO' = sin P_h

  Thus, sin P_a = sin P_h cos α

  Since angles are very small; P_a= P_h cos α=8.8''cos α

- The values of horizontal parallax are listed in the nautical almanac. The average value of P_h is 8.8''.
- α is the altitude and P_a is the parallax in altitude.



(b) Correction for parallax

➢ Correction for semi-diameter

(a) Correction in Vertical Angle (Altitude)

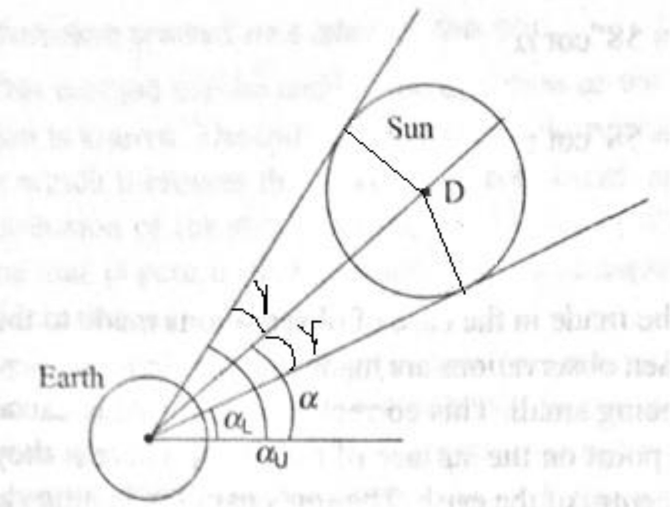
- While observing the Sun, it is not possible to bisect the centre of the body.

- As shown in the fig. the semi-diameter is one half of the angle subtended at the centre of the Earth by the diameter of the Sun (2γ).

- As the distance of the Earth from the Sun varies over the year, the semi-diameter does not remain constant and varies from 15 ' 45" to 16' 18".

- The values are listed in the nautical almanac.

- The Sun is observed by touching the upper or lower end of its diameter with the cross hairs, as the centre of the Sun can not be observed.



(c) Correction for semidiameter

- If the upper end is observed and made tangential, the altitude reading obtained is corrected by subtracting i. e. $\alpha = \alpha_U - \gamma$. If lower end is observed the correct altitude will be $\alpha = \alpha_L + \gamma$. If average of the upper and lower observation is taken no correction is required ($\alpha = (\alpha_U + \alpha_L)/ 2$).

➢ Correction in horizontal angle (azimuth)

- Fig. shows the elevation and plan view of horizontal measurement to the Sun. The correction to the observed horizontal angle is given by:

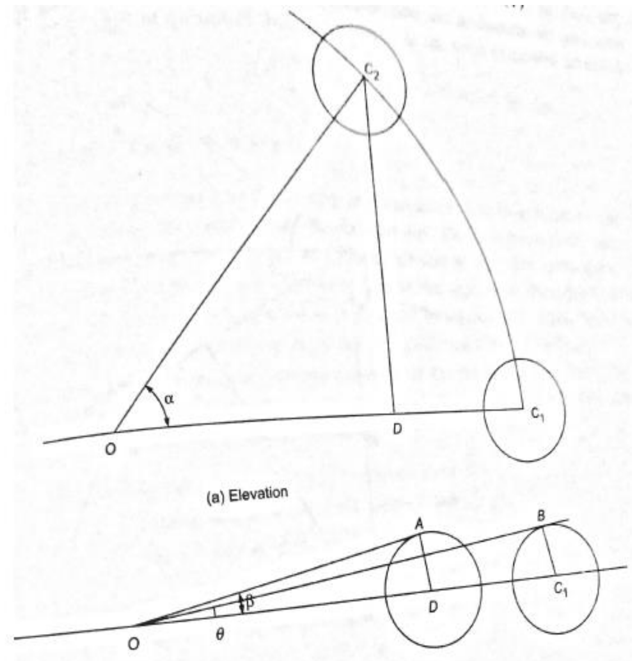
  c = γ sec α

  γ= one half of the angle subtended at the centre of the Earth by the diameter of the Sun or Sun's semi-angle

  α= corrected altitude of the Sun

- Proof:

  Let r be the radius of the Sun



(b) Plan

$$From\ fig.(b)$$

$$\sin\beta = \frac{r}{OD} \quad and \quad \sin\theta = \frac{r}{OC_1}$$

$$Since\ \beta\ and\ \theta\ are\ small\ angles,$$

$$\beta = \frac{r}{OD} \quad and \quad \theta = \frac{r}{OC_1}$$

$$So,\ \beta = \frac{1}{OD}(OC_1)\theta = \frac{OC_1}{OD}\theta$$

$$From\ fig.(a)$$

$$\cos\alpha = \frac{OD}{OC_2} = \frac{OD}{OC_1}$$

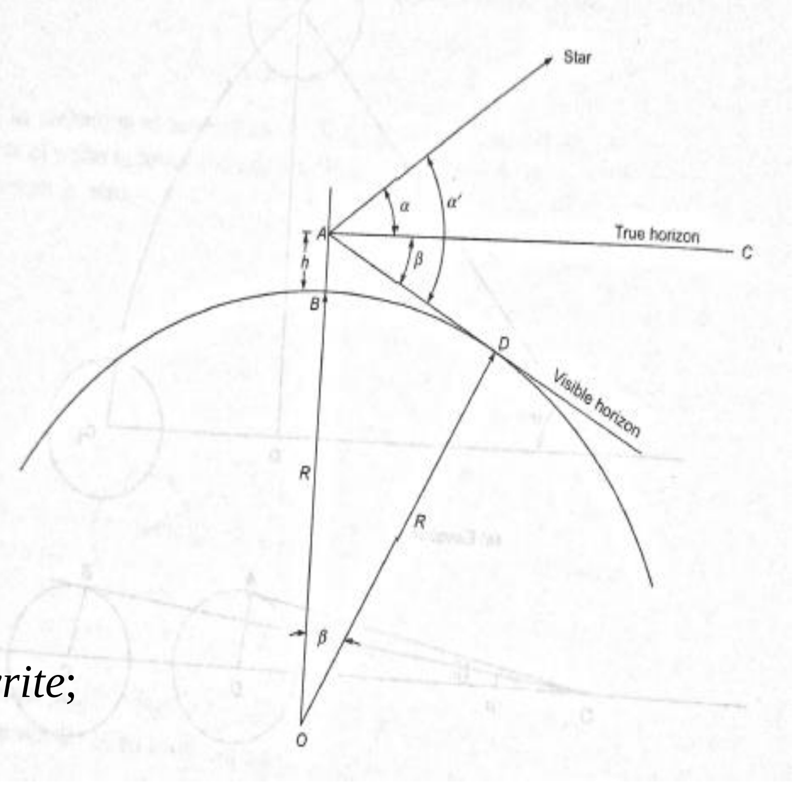
$$Thus\ \beta = \theta\sec\alpha\ or\ \gamma\sec\alpha\ (Since\ \theta = \gamma\ Sun's\ semi-angle)$$

$$or\ correction,\ \ c = \beta = \gamma\sec\alpha$$

➢ Correction for dip

- Angle of dip is the angle between true horizon and visible horizon.
- If a sextant is used for measuring the altitude of the star, measurements are from visible horizon. Hence angle of dip should be subtracted from the altitudes measured.

  Let R be the radius of the Earth. From the fig.



$$AD = \sqrt{AO^2 - DO^2}$$

$$= \sqrt{(R+h)^2 - R^2} = \sqrt{2hR + h^2}$$

$$\tan\beta = \frac{AD}{R} = \frac{\sqrt{2hR + h^2}}{R}$$

Since h is small compared to R, we may write;

$$\beta = \sqrt{\frac{2h}{R}}$$

$$\text{Correct angle } \alpha = \alpha' - \beta$$

Different Time Systems

- Time is the interval which lapses between any two instances.
- The time taken by one apparent revolution of the Sun about the Earth is known as a **Solar Day**
- The time taken by the Sun apparently to make a complete circuit of the ecliptic is equal to **one Tropical Year**.
- The Earth makes 366.2422 revolutions during a tropical year.
- The Sun travels through a total hour angle of 360° or 244 Hours in tropical year.
- The Sun apparently makes 365.2422 revolutions about the Earth. Hence, apparently there are 365.2422 days in a solar year.
- The time interval between successive upper transits of the vernal equinox (first point of Aries) is known as a sidereal day. Thus in a year, there are 366.2422 sidereal days. Thus;

365.2422 solar days = 366.2422 sidereal days
1 solar day = 1.0027379 sidereal days
1 sidereal day = 0.9972696 solar days

- The sidereal time is suitable for the astronomers while solar time is convenient for everyday use.

The following systems are used for measuring time:

1. Sidereal time
2. Solar apparent time
3. Mean solar time
4. Standard time

1. Sidereal Time

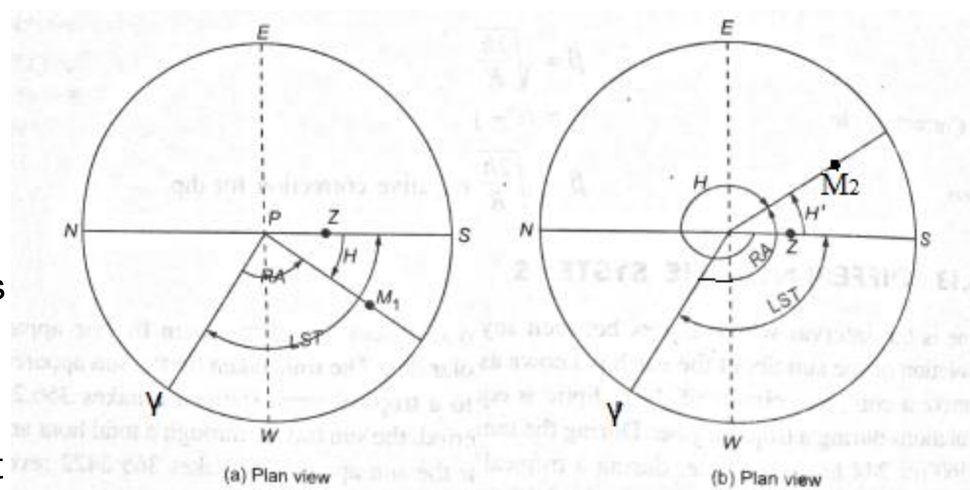
The sidereal day is the interval of time between two successive upper transits of the first point of Aries. The sidereal day is divided into 24 hours, each hour is divided into 60 minutes and each minute into 60 seconds.

Local Sidereal Time

- Local sidereal time (LST) is the time interval that has elapsed since the transit of the first point of Aries to the meridian of the observer.
- Fig. shows the plan view of celestial sphere seen from the pole and thus:

$$LST = \angle ZP\gamma$$

In fig. (a)
NPZS shows the observers meridian.
PM_1 shows Hour circle of star M_1
Thus
Star's hour angle +Star's Right Ascension = LST
In fig. (b)
LST = RA - H'
It may also be noted that when the star is on the observer's meridian LST = RA (Since hour angle is zero).



- The interval of the time between two successive lower transits of the mean sun is known as mean solar day.
- The duration of mean solar day is the average of solar days of the year.
- The mean solar day begins at the mid-night and ends on next midnight.
- The zero hour of the mean solar day is at the local mean mid-night (LMM).
- The instant when the mean sun crosses the upper transit is known as local mean noon (LMN)
- The local mean time (LMT) is given by the hour angle of the mean sun reckoned westward from 0 to 24 hours.

Standard Time

- Local mean time changes from place to place. So to avoid confusion of using different local time in a country, local mean time of a particular place is used as the standard mean time in the entire country.
- The standard meridian of a country is generally selected such that its distance from the Greenwich is in whole number of hours without any fraction.
- However, India is an exception to this as its standard meridian is 5½ hours (82° 30') east of Greenwich. It passes near Allahabad.
- Though the standard mean time is followed through in a country, but if required local mean time may be found with the formula below.

  Local mean time = Standard mean time ± difference in longitudes in hours
- Plus sign is used if the place is in east of standard meridian and minus sign is used for the places west to the standard meridian.
- World has accepted local mean time of Greenwich in U. K. as universal time (UT) or Greenwich mean time (GMT). Indian Standard Time = GMT+5½ Hours.

- In general, hour angle and RA is measured in (hour, minute, second) , then following relation may be used for conversion:

$$24 \text{ hours} = 360°$$
$$1 \text{ hour} = 15°$$
$$1 \text{ min.} = 15'$$
$$1 \text{ Sec.} = 15''$$

- The difference between the local sidereal time of two different places is equal to the difference of their longitudes expressed in terms of hours.

Solar Apparent Time

- The time based on apparent motion of the sun around the Earth is known as solar apparent time.
- The time interval between two successive lower transits of the Sun over the observer's meridian is called apparent solar day.
- The reason for selecting lower transits of the Sun is to see that the day changes only at midnight not at noon.
- The solar apparent day is not uniform throughout the year since the orbit around the Earth is not circular but elliptic and also due to the apparent diurnal path of the Sun.
- The local apparent days are longer in summer and shorter in winter.
- Due to non-uniformity of the apparent solar day, the clock cannot be used to give us solar time. Sun dials can be used to get apparent solar time.

Mean Solar Time

- To overcome the difficulty of non-uniformity of the Sun's apparent motion in recording of the time, a fictitious sun is assumed to move at a uniform rate along the equator.
- The fictitious sun is called mean sun and start and arrival of the mean sun and the true sun are assumed to be same at the vernal equinox.

➢ Relationship between difference in longitudes and time interval

360° of longitude is equal to 24 hours of time interval. Hence

360° = 24 hours
1 hour = 15°
1 min. = 15'
1 Sec. = 15"

➢ Conversion of local time to standard time

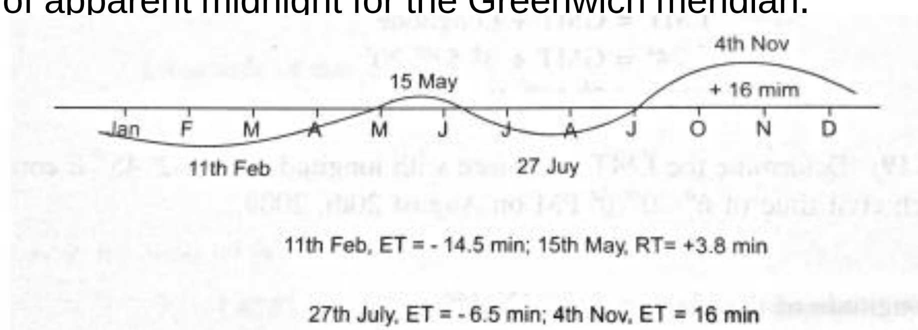
Since the apparent motion of the Sun is from east to west, local time of the place towards the east of standard meridian is more and if place is towards west local time is less.

Local time = Standard time ± Difference in longitudes (use +ve for E; -ve for West)

➢ Conversion apparent solar time to mean solar time

- The difference of apparent solar time (local apparent time, LAT) and mean solar time (local mean time, LMT) is known as equation of time, ET. Thus;

  ET = LAT – LMT
- The value of ET varies from 0 to 16 min. It is zero four times in a year.
- The Nautical Almanac gives the values of equation of time (ET) for everyday in a year. The values given are at the instant of apparent midnight for the Greenwich meridian.
- For any other day ET may be linearly interpolated.
- The ET is same for all the places.



➢ Conversion of sidereal time interval to mean solar time interval

366.2424 sidereal days = 365.2422 mean solar days

Sidereal time to mean solar time:

$$1 \text{ sidereal day} = \frac{365.2422}{366.2422}$$

$$= \left(1 - \frac{1}{366.2422}\right) \text{ mean solar day}$$

$$\text{or Sidereal day} - \text{Mean solar day} = -\frac{1}{366.2422} \text{ hours}$$

$$= -\frac{1}{366.2422} \times 24 \times 60 \times 60$$

$$= -235.909 \text{ seconds}$$

$$\text{or sidereal hour} - \text{Mean solar hour} = -\frac{235.909}{24}$$

$$= -9.8296 \text{ seconds.}$$

$$\text{or sidereal hour} = (\text{Mean solar hour} - 9.8296) \text{ seconds.}$$

Mean solar time to Sidereal time:

$$1\,\text{mean solar day} = \frac{366.2422}{365.2422} = \left(1 + \frac{1}{365.2422}\right)\text{sidereal day}$$

$$\text{or Mean solar day-Sidereal day} = \frac{1}{365.2422}\ \text{hours} = \frac{1}{365.2422} \times 24 \times 60 \times 60 = 236.555\,\text{seconds}$$

$$\text{or sidereal hour-Mean solar hour} = \frac{236.555}{24} = 9.8565\,\text{seconds.}$$

$$\text{or Mean solar hour} = (\text{sidereal hour} + 9.8565)\ \text{seconds.}$$

- **Determining local sidereal and local solar mean time**

LST = Local Sidereal Time
LMT = Local mean time
LMM = Local mean midnight (0 hr)
LMN = Local mean noon (12 hr)
GST = Greenwich sidereal time
GMT = Greenwich mean time
GMM = Greenwich mean midnight
GMN = Greenwich mean noon

1. To find LST at LMM and LMN when GST at GMM and GMN are given

- 1 mean solar hour = $1^h + 9.8565^s$ sidereal hour
- If the place is to the west of Greenwich its LMM will have LMM certain hours after GMM and will have its LMM certain hours earlier if it is to the east of Greenwich.
- LST at LMM = GST at GMM ± 9.8565^s × hour of the longitude $\left(\frac{W}{E}\right)$
- LST at LMN = GST at GMN ± 9.8565^s × hour of the longitude $\left(\frac{W}{E}\right)$

2. To find LST when LMT is given

- For this, first find the mean time interval elapsed after LMM. Then convert mean time interval into sidereal time interval by applying an acceleration at the rate of 9.8565 sec. per hour. Then

  LST at LMT = LST at LMM + sidereal time interval

  In case of GST at GMM is given, first convert it to LST at LMM as done in the first article, then use the above equation.
- In the same way: LST at LMN = LST at LMN + sidereal interval.

3. To find LMT when LST is given

- If LST at LMM is not directly given, GST at GMM is given first convert it to LST at LMM.
- Subtract LST at LMM from LST and get sidereal time interval elapsed.
- Convert the side real time interval to mean time interval by subtracting at the rate of 9.8565^s per hour to get LMT.

4. To find LMT of transition of a star when GST of GMN is given

- At the time of transit local hour angle is zero. Hence the local sidereal time is equal to the right ascension of the star.
- The right ascension and declination of all important stars may be found from nautical almanac.
- Then the LMT of upper transit of the star can be determined by the method given in article 3.
- The procedure is summarized below;

  (i) Find RA of the star from the current Nautical Almanac.

  (ii) Determine the LST of LMM (or LMN) from GST of GMM (or GMN).

(iii) Then sidereal interval that has elapsed since LMM = LST of upper transit – LST of LMM
(iv) Convert the sidereal interval to the mean time interval by applying retardation at the rate of 9.8565^s per hour
(v) Since LMT of LMM is zero, the sidereal interval calculated above is the LMT.

5. To find LMT of transit of star at any other longitude when the GMT of first point of Aries (γ) is given

Consider a place in western longitude:

- Here the transit of **γ** takes place after that at Greenwich by longitude divided by 15 hours. This is sidereal interval of time.
- Since sidereal clock gains over the mean clock, the difference between mean time clock and sidereal clock go on decreasing by 9.8296 sec. per sidereal hour. Hence:

LMT of transit of **γ** = GMT of transit of **γ** – 9.8296^s per westerly hour

For a place in eastern longitude:
LMT of transit of **γ** = GMT of transit of **γ** – 9.8296^s per westerly hour

6. To find LMT from LST, if GMT of transit of γ on the same day is also given

The following procedure may be adopted:
(i) Compute the LMT of transit from GMT as explained in the article 5.
(ii) Convert the given LST to mean time interval.
(iii) Then, LMT = LMT of transit + mean time interval

7. To find GMT at the next transit of γ from the sidereal time at GMM

- The sidereal time of a place at 0^h i.e. at GMM is the number of hours lapsed since the transit of **γ**.
- Hence next transit of **γ** will occur after 24-*s* hours, where *s* is GST at GMM. This is in sidereal hours.
- This 24-*s* sidereal hours can be converted into mean time hour to get GMT at the next transit of **γ**.

8. To find LMT of LAN on the day of given GMT of GAN (Greenwich apparent noon)

- The equation of time (ET) at the apparent noon is zero. Hence GMT of GAN is equal to ET.
- For the place west of Greenwich apparent noon occurs later. The hour of west longitude is found. The interpolation of LMT of LAN (local apparent noon) is done with the GMT of GAN data of the day and one day after i. e. with 24 hours difference.
- For the place east of Greenwich LAN occurs earlier. So to perform interpolation data of GMT of GAN of the day and one day before is required.
- Once ET is known LMT of LAN can be determined.

Example: The table shows the extract from the Nautical Almanac of a particular year. Find the LMT of LAN on May 12 at a place of longitude 123° E.

Date	GMT			Difference
	Hour (h)	Minute (m)	Second (s)	
May 10	12	3	12.22	10.8^s
May 11	12	3	23.02	10.68^s
May 12	12	3	33.70	10.54^s
May 13	12	3	44.24	

Solution:

Longitude 123° E = 123×24/360 = 8.2^h E

Since the place is to the east, the LMT is ahead of Greenwich time.

From the table

Difference between GMT of GAN of May 11 and 12 is 10.68^s for 24 hrs.

Difference for 8.2^h is = 8.2×10.68/24 = 3.65^s

LMT of LAN on May 12 = $12^h\ 03^m\ 33.70^s - 3.65^s = 12^h\ 03^m\ 30.05^s$

9. To find GST of elongation of a star

- Referring to the Fig. shown, when star is at elongation, the angle ZMP of the astronomical triangle is a right angle.
- The hour angle H of the star can be determined from the latitude of the place and the declination of the star using Napier's rule that:

Sine of middle part = product of tangents of adjacent parts

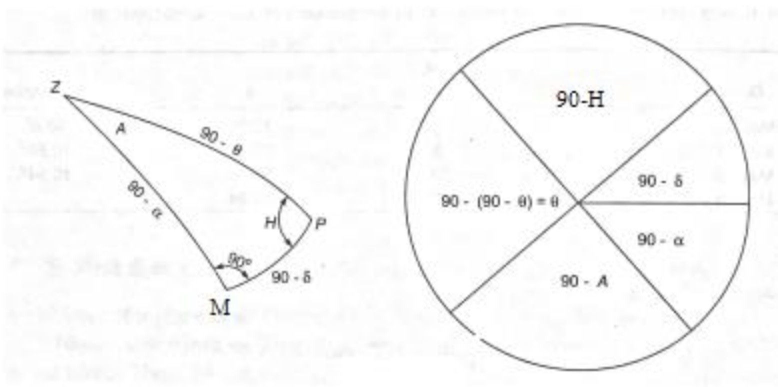
- In this case

$\sin(90-H) = \tan\theta \tan(90-\delta)$

$\cos H = \tan\theta \cot\delta$

- Using the earlier article on local sidereal time
- LST of star at elongation = Star's hour angle+ Star's right ascension
- Thus to get LST of elongation of a star,

add westerly hour angle to RA or subtract easterly angle RA of the star at its elongation. If the result is more than 24 hours, 24 hours are to be deducted and if the result is negative 24 hours are to be added.



Example: Find LST of western elongation of a star at a place of latitude 42°30´30´´ N. The RA of the star is $8^h\,26^m\,33^s$ and its declination is 78°15´0´´.

Solution: When the star is at elongation:

$$\cos H = \tan\theta \cot \delta = \tan 42°30´30´´ / \tan 78°15´0´´ = 0.19065282$$

$$H = 79.009115°$$

$$= 79°00´32.82´´$$

$$= 5^h\,16^m\,2.19^s$$

$$LST = 5^h\,16^m\,2.19^s + 8^h\,26^m\,33^s = 13^h\,42^m\,35.19^s$$

Nautical Almanac

- There is an informative table of book annually known as **Nautical Almanac** that gives the value of equation of time (ET).
- The ET varies in magnitude throughout the year.
- Its value is published in the **Nautical Almanac** at the instant of apparent midnight for Greenwich meridian for each day of year.

Practice Problems:

1. Express the following difference in longitudes into interval of time.
   (i) 46°26´30´´ (ii) 232°30´42´´
2. Convert the following interval of time into difference of longitudes:
   (i) $5^h\,36^m\,30^s$
   (ii) $16^h\,29^m\,15^s$
3. The standard meridian of Indian time is 82°30´ E. Determine local mean time for the places with longitudes (a) 16° E (b) 18° W at the instant of Indian standard time $20^h\,20^m\,12^s$.

4. Determine Greenwich mean time (GMT) corresponding to the following LMT:
   (a) $8^h\ 12^m\ 16^s$ AM at 38°45´ W longitude
   (b) $6^h\ 8^m\ 24^s$ AM at 58°20´ E longitude
5. Determine the LMT of a place with longitude 115° 42´ 45´´ E corresponding to the Greenwich civil time of $6^h\ 30^m\ 0^s$ PM on August 20th 2009.
6. Find the equation of time at 16 hrs GMT on April, 2010 from the following data:
   On April 12, 2010; ET = -3 m 52.4 s
   On April 13, 2010; ET = -3 m 32.8 s.
7. Find LMT of observation at a place from the following data:
   Longitude of the place = 22° W
   ET at Greenwich Mean at Noon = 8m 22.5 s
   Rate of change of ET = +0.21 s per hour
   LAT (Local Apparent Time) of observation = 18 hours.
8. Find the LAT at a place in longitude 48° 30´ E corresponding to LMT $8^h\ 30^m\ 15^s$. The equation of time of GMN (Greenwich Mean Noon) is $4^m\ 48^s$ additive and decreasing at the rate of 0.28^s per hour.
9. Convert $6^h\ 20^m\ 30^s$ sidereal time interval to mean solar time interval.
10. Convert $8^h\ 25^m\ 30^s$ mean solar time to sidereal time.
11. If GST of GMM on a certain day is $14^h\ 25^m\ 42^s$, determine the LST of LMM at a place in longitude (a) 150° 30´ 30´´ W (b) 150° 30´ 30´´.
12. Find LST at a place 66° W at 10.00 AM local mean time if GST at GMM is $16^h\ 30^m\ 20^s$.
13. Find the LST at a place 80° E at 6 PM, if GST at GMM is $5^h\ 30^m\ 30^s$.
14. Find LMT at a place 90° W, if LST is $14^h\ 38^m\ 30^s$, given that GST at GMM is $7^h\ 12^m\ 24^s$.
15. Determine the LMT of upper and lower transit of a star at a place 160° W whose RA is $20^h\ 20^m\ 20^s$, if GST of GMN is $11^h\ 45^m\ 20^s$.
16. On a particular day the GMT of transit of the first point of Aries γ is $12^h\ 20^m\ 30^s$. Find the LMT of transit of γ on the same day at a place (a) longitude of 63° W (b) 48° E.
17. The local sidereal time at a place 54° E is $10^h\ 20^m\ 45^s$. Find the corresponding LMT given that the GMT of transit of γ on the same day is $5^h\ 25^m\ 30^s$.