



Constraint Satisfaction (Cryptarithmic & Means End Analysis)

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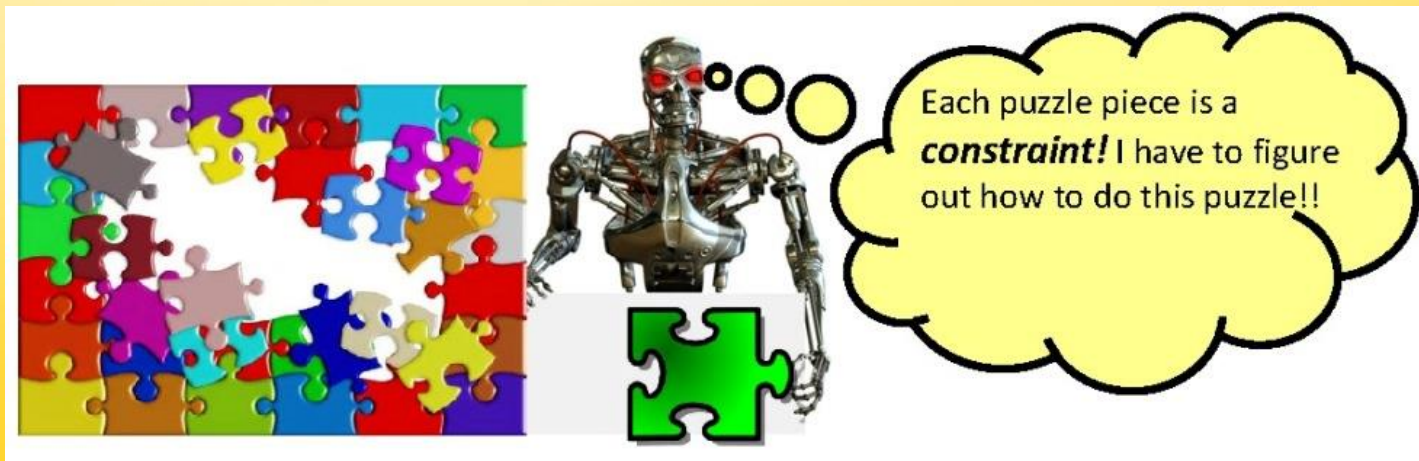
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Constraint Satisfaction

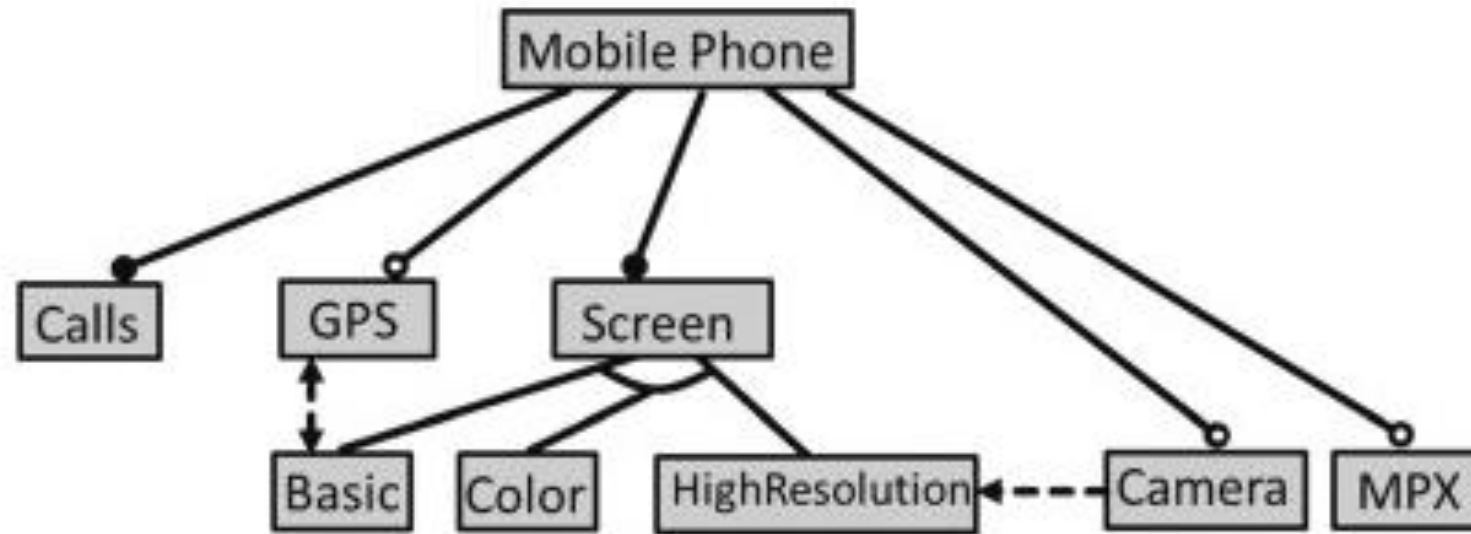
- Many problems in AI can be considered as problems of constraint satisfaction, in which the goal state satisfies a given set of constraint
- **constraint satisfaction** is the process of finding a solution to a set of constraints that impose conditions that the variables must satisfy
- A solution is therefore a set of values for the variables that satisfies all constraints
- Problems that can be expressed as constraint satisfaction problems are the eight queens puzzle, the Sudoku solving problem and many other logic puzzles, the Boolean satisfiability problem, scheduling problems, bounded-error estimation problems and various problems on graphs such as the graph coloring problem



HISTORY

- Constraint satisfaction originated in the field of artificial intelligence in the 1970s ((Laurière 1978))
- During the 1980s and 1990s, embedding of constraints into a programming language were developed
- Languages often used for constraint programming are Prolog and C++





- *The general form of the constraint satisfaction procedure is as follows:*
- Until a complete solution is found or until all paths have led to lead ends, do
- 1. select an unexpanded node of the search graph.
- 2. Apply the constraint inference rules to the selected node to generate all possible new constraints.
- 3. If the set of constraints contains a contradiction, then report that this path is a dead end.
- 4. If the set of constraints describes a complete solution then report success.
- 5. If neither a constraint nor a complete solution has been found then apply the rules to generate new partial solutions. Insert these partial solutions into the search graph

Cryptarithmic Problem

- Cryptarithmic Problem is a type of constraint satisfaction problem where the game is about digits and its unique replacement either with alphabets or other symbols
- In cryptarithmic problem, the digits (0-9) get substituted by some possible alphabets or symbols
- The task in cryptarithmic problem is to substitute each digit with an alphabet to get the result arithmetically correct.

Can You Solve this



$$\begin{array}{r} \text{S E N D} \\ + \text{M O R E} \\ \hline \text{M O N E Y} \end{array}$$

- **SEND + MORE = MONEY**

- **CONSTRAINTS:-**

- 1. no two digit can be assigned to same letter.
- 2. only single digit number can be assign to a letter.

- Initial state of problem.

- D=? E=? Y=? N=? R=? O=? S=? M=?

- C1=? C2=? C3=?

- C1 ,C 2, C3 stands for the carry variables respectively.

- Goal State: the digits to the letters must be assigned in such a manner so that the sum is satisfied

$$\begin{array}{r} \text{SEND} \\ + \text{MORE} \\ \hline \text{MONEY} \\ \hline \end{array}$$

Starting from the left hand side (L.H.S) , the terms are S and M.
Assign a digit which could give a satisfactory result. Let's
assign S->9 and M->1

Hence, we get a satisfactory result by adding up the terms and
got an assignment for O as O->0 as well.

$$\begin{array}{r} S \\ + M \\ \hline MO \end{array} \longrightarrow \begin{array}{r} 9 \\ + 1 \\ \hline 10 \end{array}$$

Now, move ahead to the next terms E and O to get N as its
output

Adding E and O, which means $5+0=0$, which is not possible
because according to cryptarithmic constraints, we cannot
assign the same digit to two letters. So, we need to think more
and assign some other value.

$$\begin{array}{r} E \\ + O \\ \hline N \end{array} \xrightarrow{\times} \begin{array}{r} 5 \\ + 0 \\ \hline 5 \end{array}$$

Note: When we will solve further, we will get one carry, so after applying it, the answer will be satisfied

$$\begin{array}{r} \mathbf{E} \\ + \mathbf{0} \\ \hline \mathbf{N} \end{array} \longrightarrow \begin{array}{r} \textcircled{1} \text{ carry} \\ \mathbf{5} \\ + \mathbf{0} \\ \hline \mathbf{6} \end{array}$$

Further, adding the next two terms **N** and **R** we get,

$$\begin{array}{r} \mathbf{N} \\ + \mathbf{R} \\ \hline \mathbf{E} \end{array} \xrightarrow{\times} \begin{array}{r} \mathbf{6} \\ + \mathbf{8} \\ \hline \mathbf{14} \end{array}$$

But, we have already assigned **E**->**5**. Thus, the above result does not satisfy the values because we are getting a different value for **E**. So, we need to think more

Again, after solving the whole problem, we will get a carryover on this term, so our answer will be satisfied, where 1 will be carry forward to the above term

$$\begin{array}{r}
 \text{N} \\
 + \text{R} \\
 \hline
 \text{E}
 \end{array}
 \qquad
 \begin{array}{r}
 \text{6} \\
 + \text{8} \\
 \hline
 \text{15}
 \end{array}$$

1 ← carry

Again, on adding the last two terms, i.e., the rightmost terms **D** and **E**, we get **Y** as its result, where 1 will be carry forward to the above term

$$\begin{array}{r}
 \text{D} \\
 + \text{E} \\
 \hline
 \text{Y}
 \end{array}
 \qquad
 \begin{array}{r}
 \text{7} \\
 + \text{5} \\
 \hline
 \text{12}
 \end{array}$$

Keeping all the constraints in mind, the final resultant is as follows

S	9
E	5
N	6
D	7
M	1
O	0
R	8
Y	2

$$\begin{array}{r}
 \text{C3(0) C2(1) C1(1)} \\
 \text{S(9) E(5) N(6) D(7)} \\
 + \text{M(1) O(0) R(8) E(5)} \\
 \hline
 \text{M(1) O(0) N(6) E(5) Y(2)}
 \end{array}$$

Practice other Cryptarithmic Problems

BASE
+BALL

GAMES

→

B	7
A	4
S	8
E	3
L	5
G	1
M	9

YOUR
+YOU

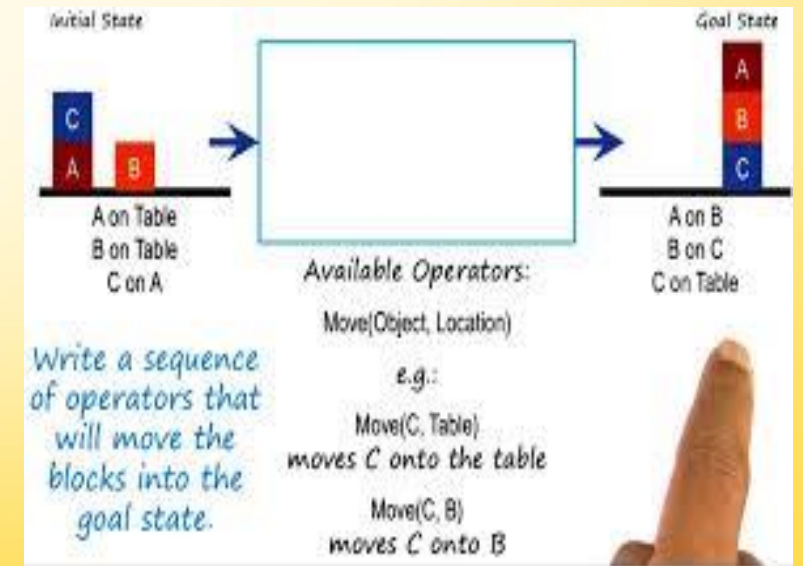
HEART

→

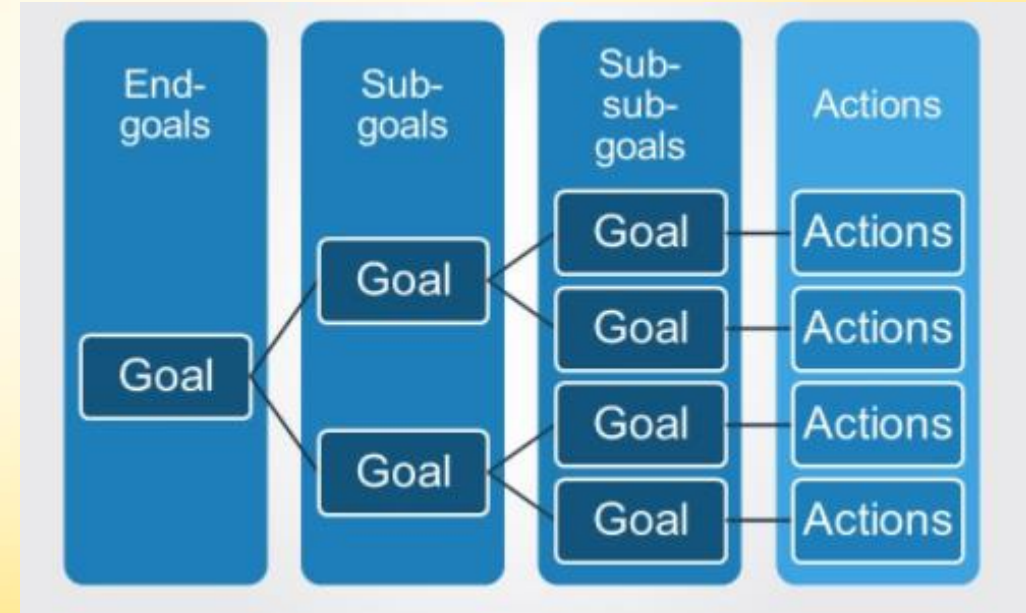
Y	9
O	4
U	2
R	6
H	1
E	0
A	3
T	8

Means End Analysis

- Most of the search strategies either reason forward or backward
- However, often a mixture of the two directions is appropriate
- Such mixed strategy would make it possible to solve the major parts of problem first and solve the smaller problems that arise when combining them together. Such a technique is called "Means - Ends Analysis".
- The means -ends analysis process centers around finding the difference between current state and goal state.
- The problem space of means - ends analysis has an initial state and one or more goal state, a set of operators with a set of preconditions their application and difference functions that computes the difference between two states



- *A problem is solved using means – ends analysis by*
- 1. Computing the current state s_1 to a goal state s_2 and computing their difference D_{12} .
- 2. Satisfy the preconditions for some recommended operator OP is selected, then to reduce the difference D_{12} .
- 3. The operator OP is applied if possible.
 - If not the current state is solved and a goal is created and means- ends analysis is applied recursively to reduce the sub goal.
- 4. If the sub goal is solved, state is restored and work resumed on the original problem
- The first AI program to use means - ends analysis was the GPS General problem solver



Algorithm

- **Step 1:** Measure the current state of things by doing as is the study and capture the status at a macro level and to a possible micro level.
- **Step 2:** Capture the deficiency in the current state and avenues for improvements (wish list) and define the goal state (to-be state). Define the to-be state at a macro level and to a possible micro level.
- **Step 3:** Compare the Current state and Goal state, and if they are at the same level, the problem is resolved.
- **Step 4:** List the differences between the current state and goal state at macro and micro levels.
- **Step 5:** Convert the differences into deletions/modifications to current state and new additions.
- **Step 6:** Define the action to implement the changes as defined in step-5.
- **Step 7:** Implement the changes and measure the actual results with the planned goals.
- **Step 8:** Do course correction and achieve the final goal.

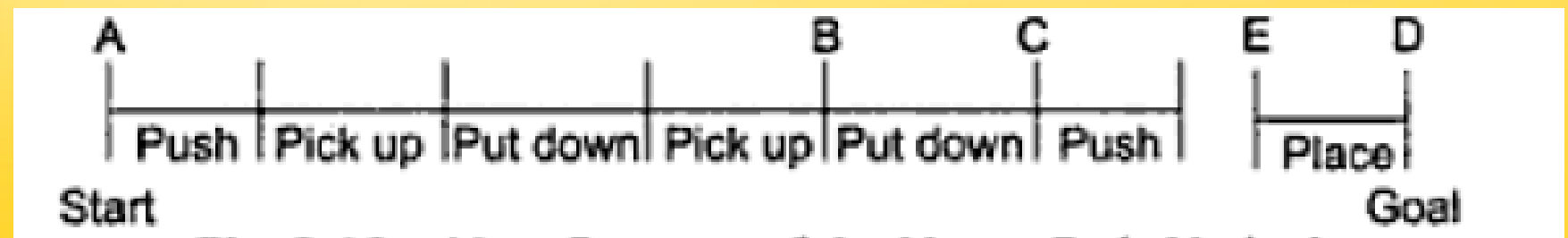
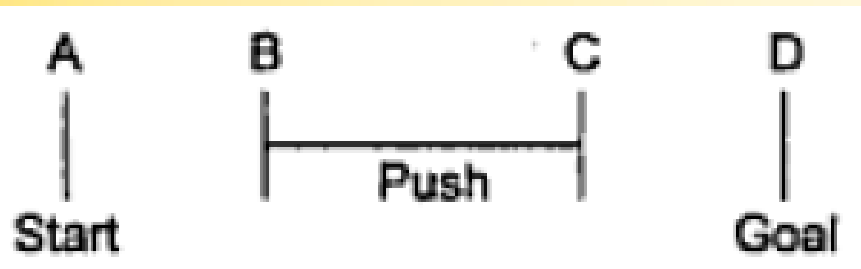
Example

- A robot was given the problem of moving a desk with two things on it from one room to another
- The objects on top must be moved
- The main difference between the start and goal would be the location of the desk

<i>Operator</i>	<i>Preconditions</i>	<i>Results</i>
PUSH(obj, loc)	at(robot, obj)^ large(obj)^ clear(obj)^ armempty	at(obj, loc)^ at(robot, loc)
CARRY(obj, loc)	at(robot, obj)^ small(obj)	at(obj, loc)^ at(robot, loc)
WALK(loc)	none	at(robot, loc)
PICKUP(obj)	at(robot, obj)	holding(obj)
PUTDOWN(obj)	holding(obj)	\neg holding(obj)
PLACE(obj1, obj2)	at(robot, obj2)^ holding(obj1)	on(obj1, obj2)

	Push	Carry	Walk	Pickup	Putdown	Place
Move object	*	*				
Move robot			*			
Clear object				*		
Get object on object						*
Get arm empty					*	*
Be holding object				*		

A Difference Table



- Means- ends analysis is useful for many human planning activities.
- Consider the example of planning for an office worker.
- Suppose we have a different table of three rules:
 - 1. If in our current state we are hungry , and in our goal state we are not hungry , then either the "visit hotel" or "visit Canteen " operator is recommended.
 - 2. If in our current state we do not have money , and if in our goal state we have money, then the "Visit our bank" operator or the "Visit secretary" operator is recommended.
 - 3. If in our current state we do not know where something is , need in our goal state we do know, then either the "visit office enquiry" , "visit secretary" or "visit co worker " operator is recommended.

