Fuzzy Classification

Fuzzy Logic

When did you come to the class?

How do you teach driving to your friend

Linguistic Imprecision, *Vagueness*, *Fuzziness* – **Unavoidable**

It is **beyond that:** What is your height ?

5 ft. 8.25 in. !!

Subject to precision of the measuring instrument – Close to 5ft. 8.25 in.

Types of Rules

Mamdani Assilian Model

R1: If x is A_1 and y is B_1 then z is C_1

R2: If x is A_2 and y is B_2 then z is C_2

A_i , B_i and $C_{i,}$ are fuzzy sets defined on the universes of x, y, z respectively

Takagi-Sugeno Model

R1: If x is A_1 and y is B_1 then $z = f_1(x,y)$

R1: If x is A_2 and y is B_2 then $z = f_2(x,y)$

For example: $f_i(x,y)=a_i x+b_i y+c_i$

Types of Rules (Contd)

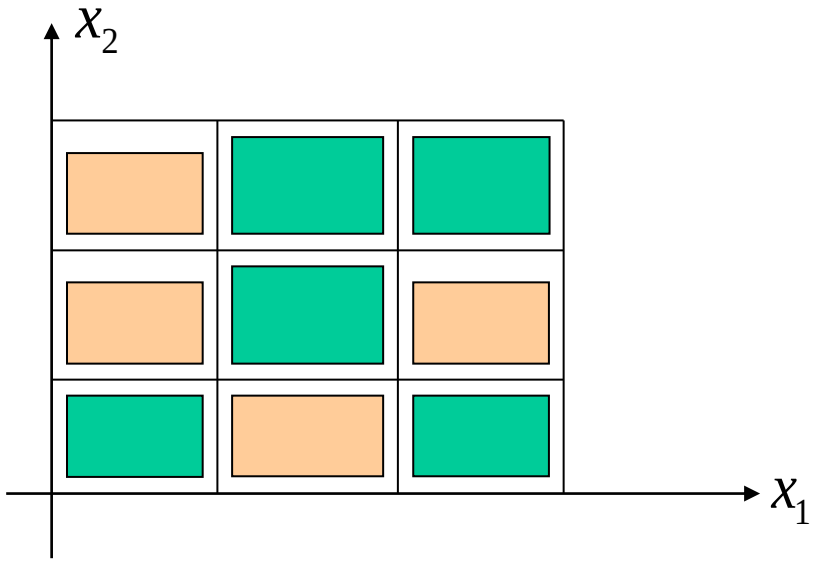
Classifier Model

R1: If x is A_1 and y is B_1 then *class* is 1

R2: If x is A_2 and y is B_2 then *class* is 2

Rule-Based Classifier

Idea: Nonlinear Partition of Feature Space



Rule 1: If x_1 is S and x_2 is S, then class 1

Rule 2: If x_1 is S and x_2 is not S, then class 2

Rule 3: If x_1 is M and x_2 is S, then class 2

Rule 4: If x_1 is M and x_2 is not S, then class 1

Rule 5: If x_1 is not S and x_2 is L, then class 1

Rule 6: If x_1 is L and x_2 is S, then class 1

Rule 7: If x_1 is L and x_2 is M, then class 2

How to find the rule from sample data.

- Project the labeled training data, and design membership functions
- Fuzzy clustering and projection to obtain membership function

Fuzzy Classification

K- nearest neighbor algorithm: Voting on crisp labels

Class 1

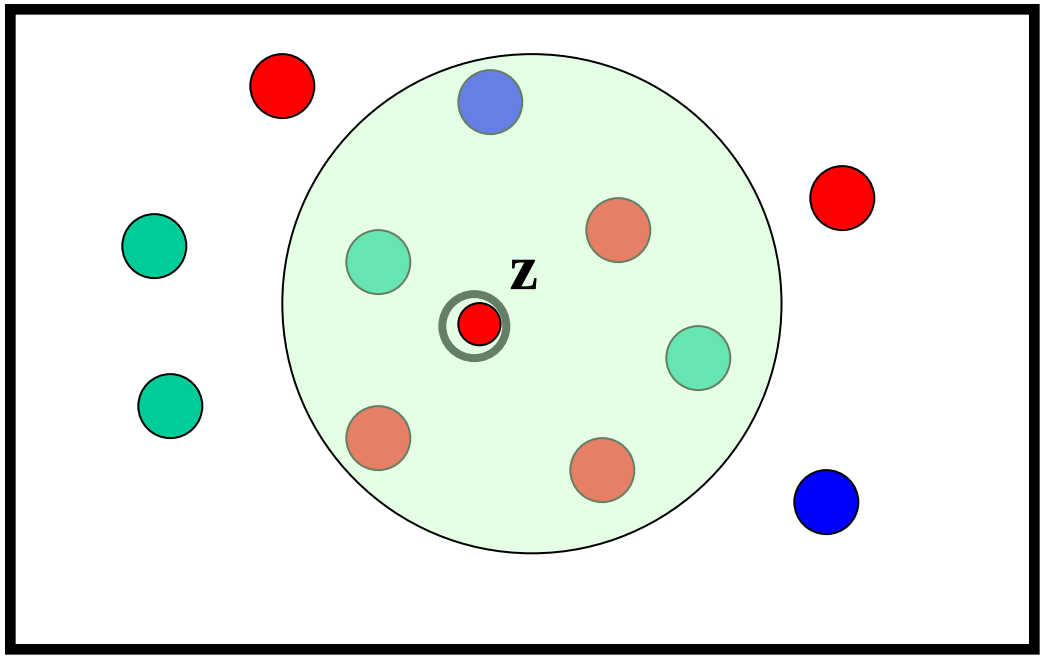
$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Class 2

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Class 3

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



K-nn Classification (continued)

The crisp K-nn rule can be generalized to generate fuzzy labels.

Take the average of the class labels of each neighbor:

$$D(\mathbf{z}) = \frac{2\begin{pmatrix}1\\0\\0\end{pmatrix} + 3\begin{pmatrix}0\\1\\0\end{pmatrix} + 1\begin{pmatrix}0\\0\\1\end{pmatrix}}{6} = \begin{pmatrix}0.33\\0.50\\0.17\end{pmatrix}$$

This method can be used in case the vectors have fuzzy or possibilistic labels also.

K-nn Classification (continued)

Suppose the six neighbors of **z** have fuzzy labels as:

$$\mathbf{x}_1 \begin{pmatrix} 0.9 \\ 0.0 \\ 0.1 \end{pmatrix} \quad \mathbf{x}_2 \begin{pmatrix} 0.9 \\ 0.1 \\ 0.0 \end{pmatrix} \quad \mathbf{x}_3 \begin{pmatrix} 0.3 \\ 0.6 \\ 0.1 \end{pmatrix} \quad \mathbf{x}_4 \begin{pmatrix} 0.03 \\ 0.95 \\ 0.02 \end{pmatrix} \quad \mathbf{x}_5 \begin{pmatrix} 0.2 \\ 0.8 \\ 0.0 \end{pmatrix} \quad \mathbf{x}_6 \begin{pmatrix} 0.3 \\ 0.0 \\ 0.7 \end{pmatrix}$$

$$\mathrm{D}(\mathbf{z}) = \frac{\begin{pmatrix} 0.9 \\ 0.0 \\ 0.1 \end{pmatrix} + \begin{pmatrix} 0.9 \\ 0.1 \\ 0.0 \end{pmatrix} + \begin{pmatrix} 0.3 \\ 0.6 \\ 0.1 \end{pmatrix} + \begin{pmatrix} 0.03 \\ 0.95 \\ 0.02 \end{pmatrix} + \begin{pmatrix} 0.2 \\ 0.8 \\ 0.0 \end{pmatrix} + \begin{pmatrix} 0.3 \\ 0.0 \\ 0.7 \end{pmatrix}}{6} = \begin{pmatrix} 0.44 \\ 0.41 \\ 0.15 \end{pmatrix}$$

Fuzzy Rule Based Classifiers

Rule1:

If x is CLOSE to a_1 and y is CLOSE to b_1 then (x,y) is in class is 1

Rule 2:

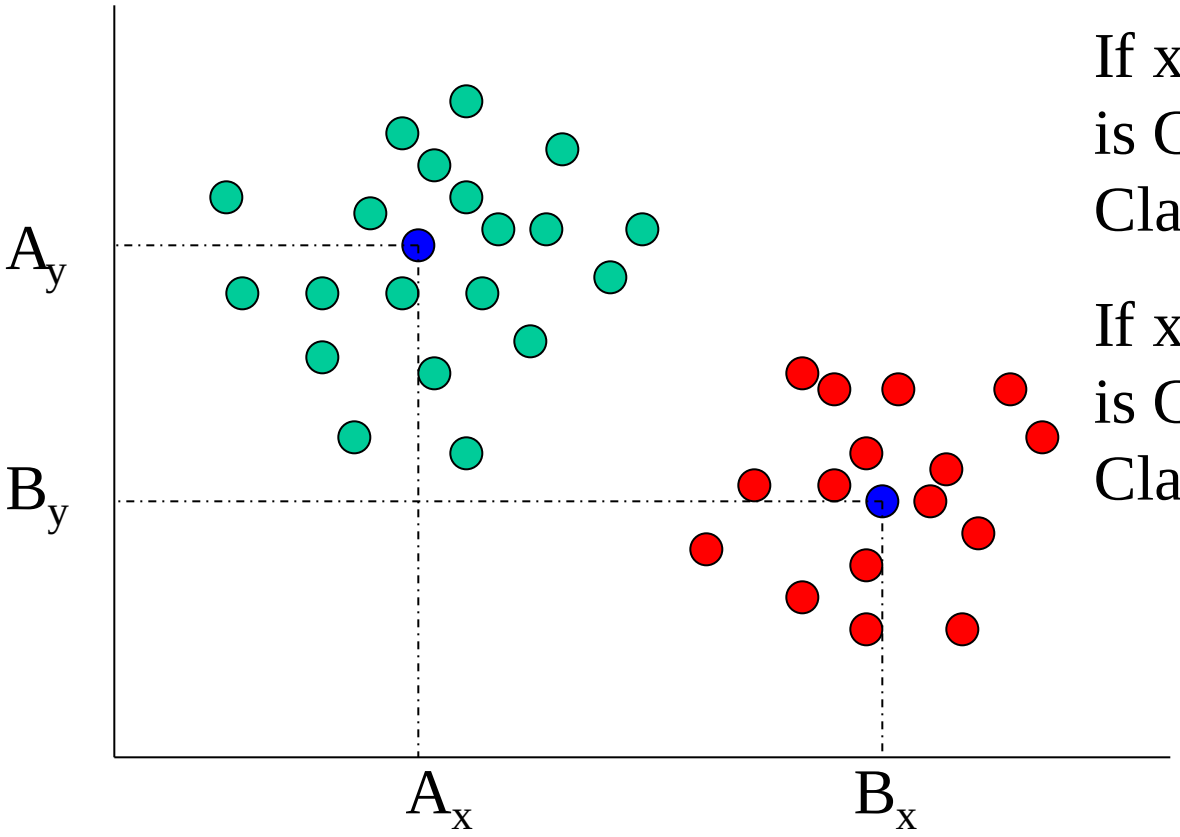
If x is CLOSE to a_2 and y is CLOSE to b_2 then (x,y) is in class is 2

How to get such rules!!

An expert may provide us with classification rules.

We may extract rules from training data.

Clustering in the input space may be a possible way to extract initial rules.



If x is CLOSE TO A_x & y is CLOSE TO A_y Then Class is

If x is CLOSE TO B_x & y is CLOSE TO B_y Then Class is

Why not make a system which learns linguistic rules from input output data.

A neural network can learn from data.

But we cannot extract linguistic (or other easily interpretable) rules from a trained network.

Can we combine these to paradigms?

YES!!

Neuro-Fuzzy Systems

Neural Networks are “Black Boxes”

Interpretation of its Internal parameters
are difficut – Not possible in many cases
(NOT Readable)

But they HAVE learning and
Generalization Abilities

Fuzzy Systems are highly interpretable in terms of fuzzy rules.

But they do not as such have learning and/or generalization abilities

Integration of these two systems leads to better systems: **Neuro-Fuzzy Systems**

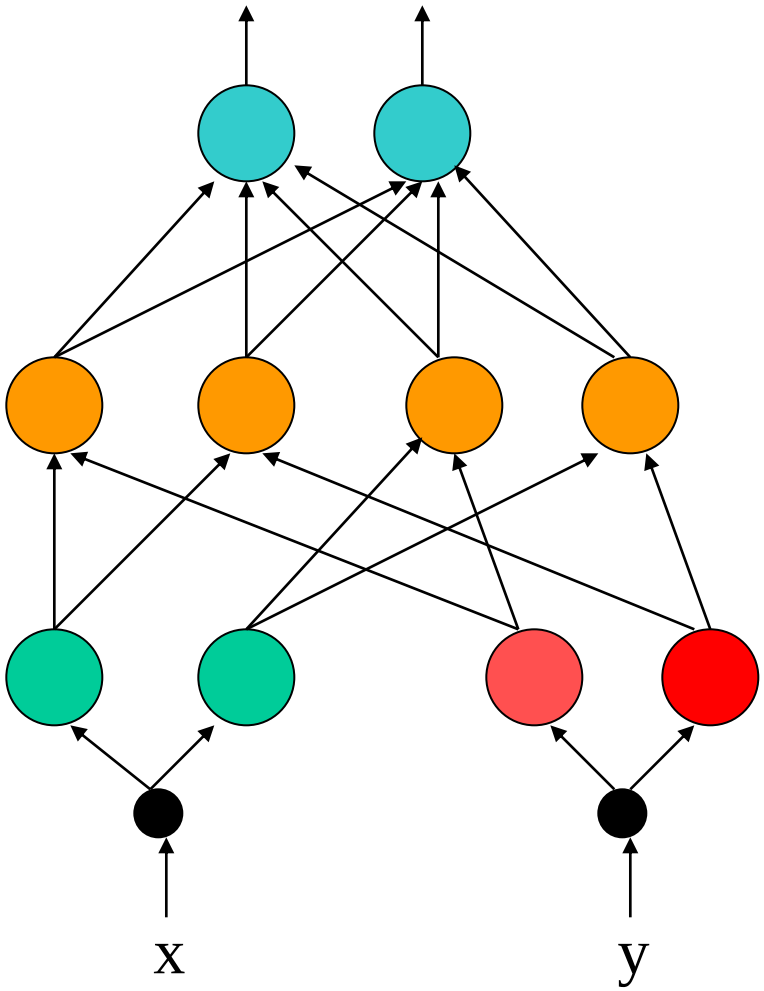
Types of Neuro-Fuzzy Systems

Neural Fuzzy Systems

Fuzzy Neural Systems

Cooperative Systems

A neural fuzzy system for Classification



Output Nodes

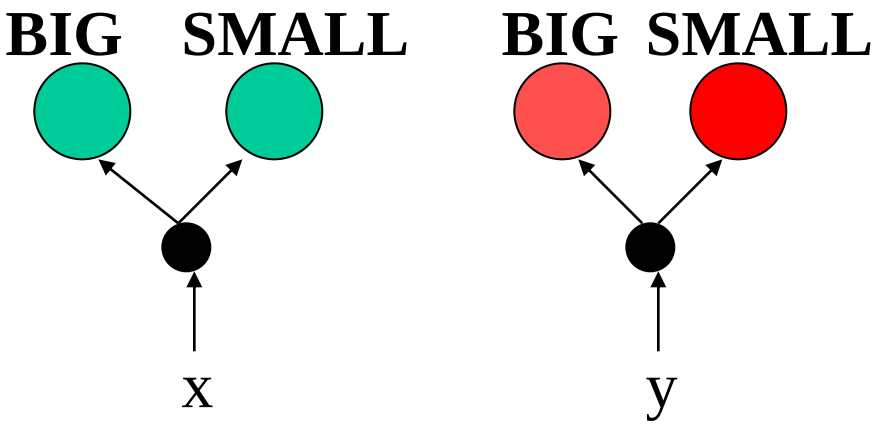
Antecedent Nodes

Fuzzification Nodes

Fuzzification Nodes

Represents the term sets of the features.

If we have two features *x* and *y* and two linguistic variables defined on both of it say BIG and SMALL. Then we have 4 fuzzification nodes.



We use Gaussian Membership functions for fuzzification ---

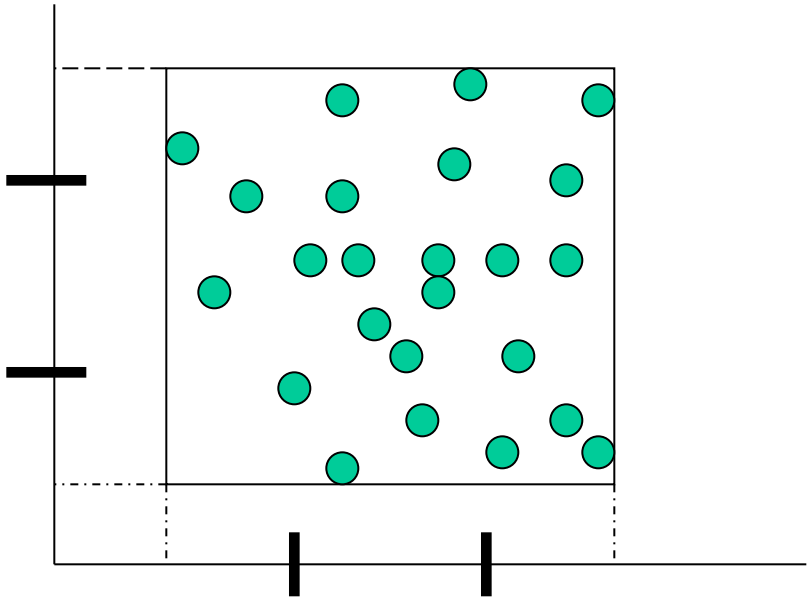
They are differentiable, triangular and trapezoidal membership functions are NOT differentiable.

Fuzzification Nodes (Contd.)

$$z = \exp\left\{-\frac{(x-\mu)^2}{\sigma^2}\right\}$$

μ and σ are two free parameters of the membership functions which needs to be determined

How to determine μ and σ

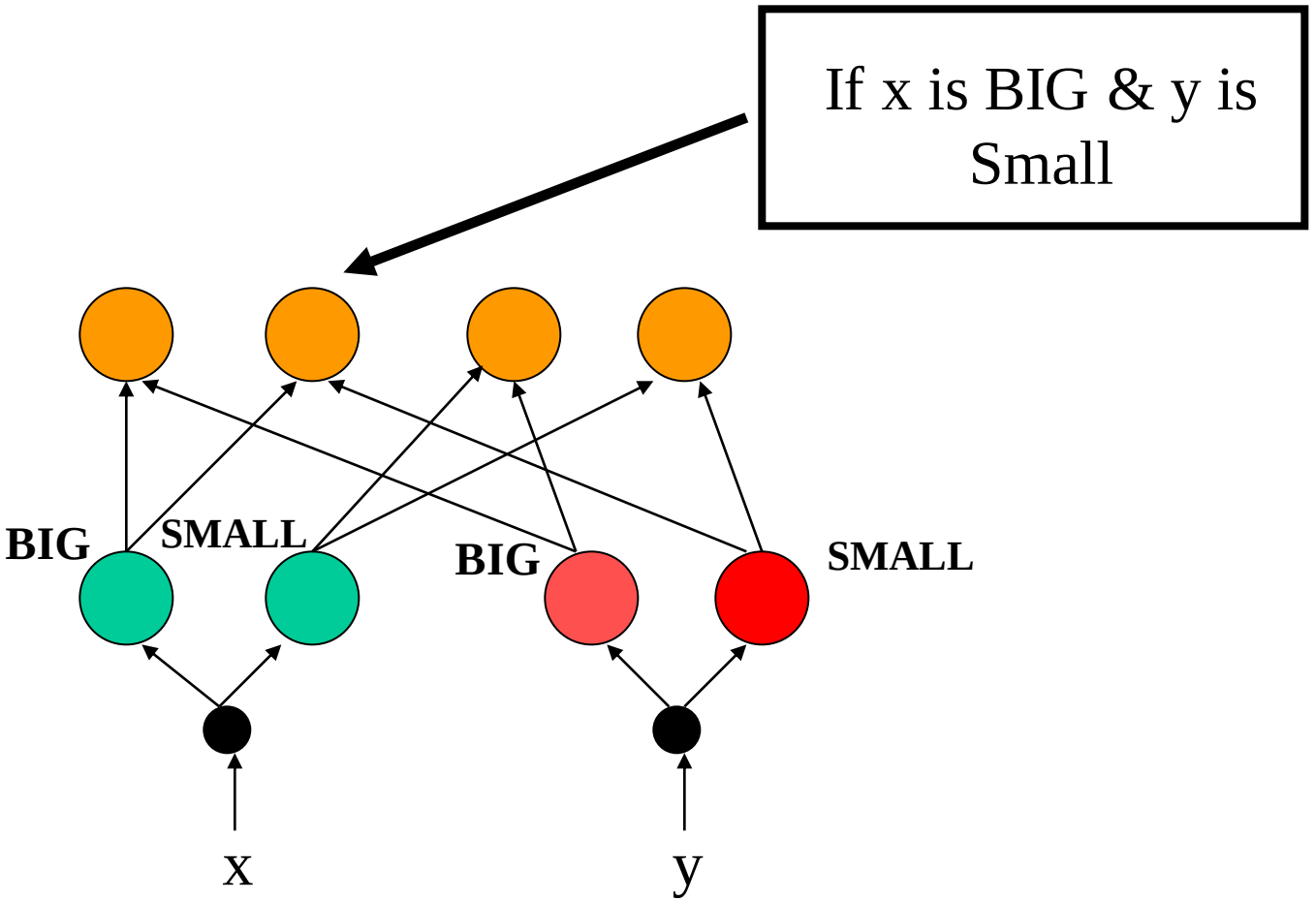


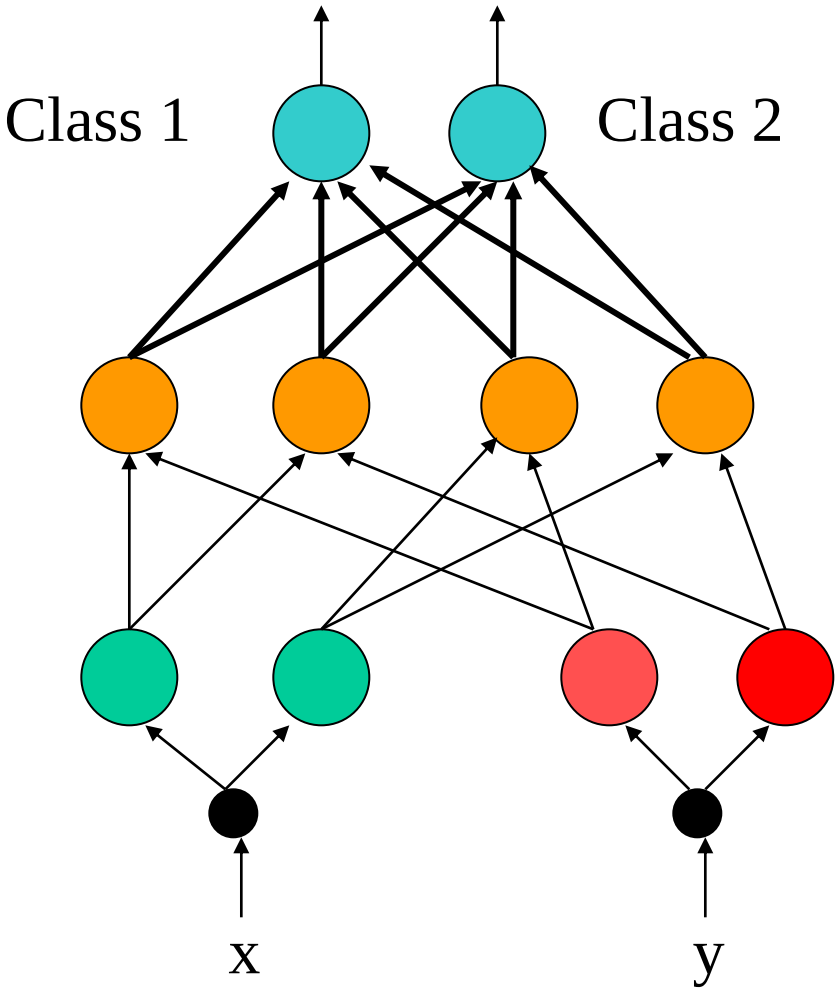
Two strategies:

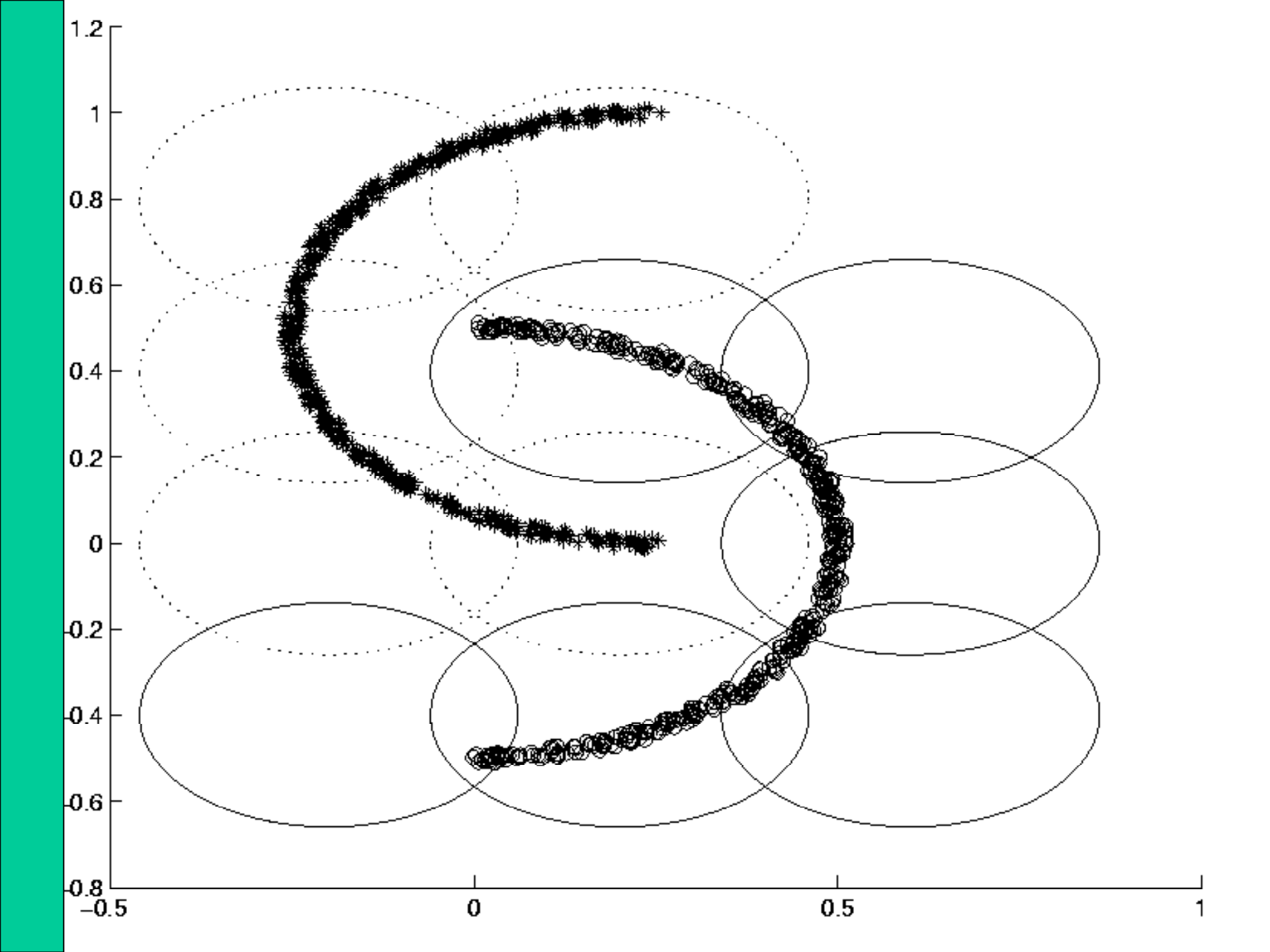
1) Fixed μ and σ

2) Update μ and σ , through any tuning algorithm

Antecedent nodes




Class 1
Class 2
x
y


1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-0.5
0
0.5
1

Further Readings

1) Neural Networks, a comprehensive foundation, **Simon Haykin**, 2nd ed. Prentice Hall
2) Introduction to the theory of neural computation, **Hertz**, **Krog** and **Palmer,** Addision Wesley
3) Introduction to Artificial Neural Systems, **J. M. Zurada**, West Publishing Company
4) Fuzzy Models and Algorithms for Pattern Recognition and Image Processing, **Bezdek**, **Keller**, **Krishnapuram**, **Pal,** Kluwer Academic Publishers
5) Fuzzy Sets and Fuzzy Systems, **Klir** and **Yuan**
6) Pattern Classification**, Duda, Hart and Stork**

Thank You